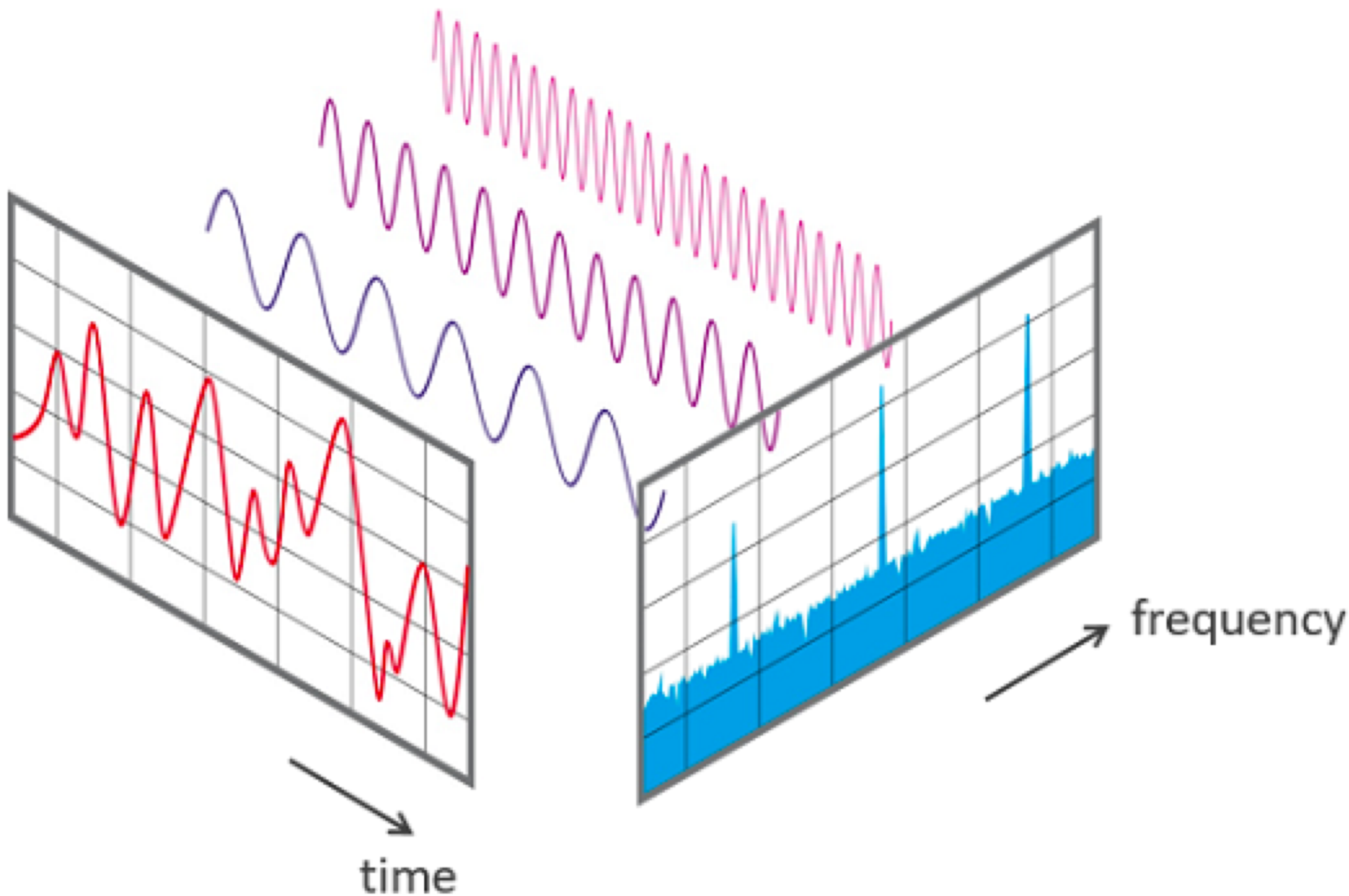
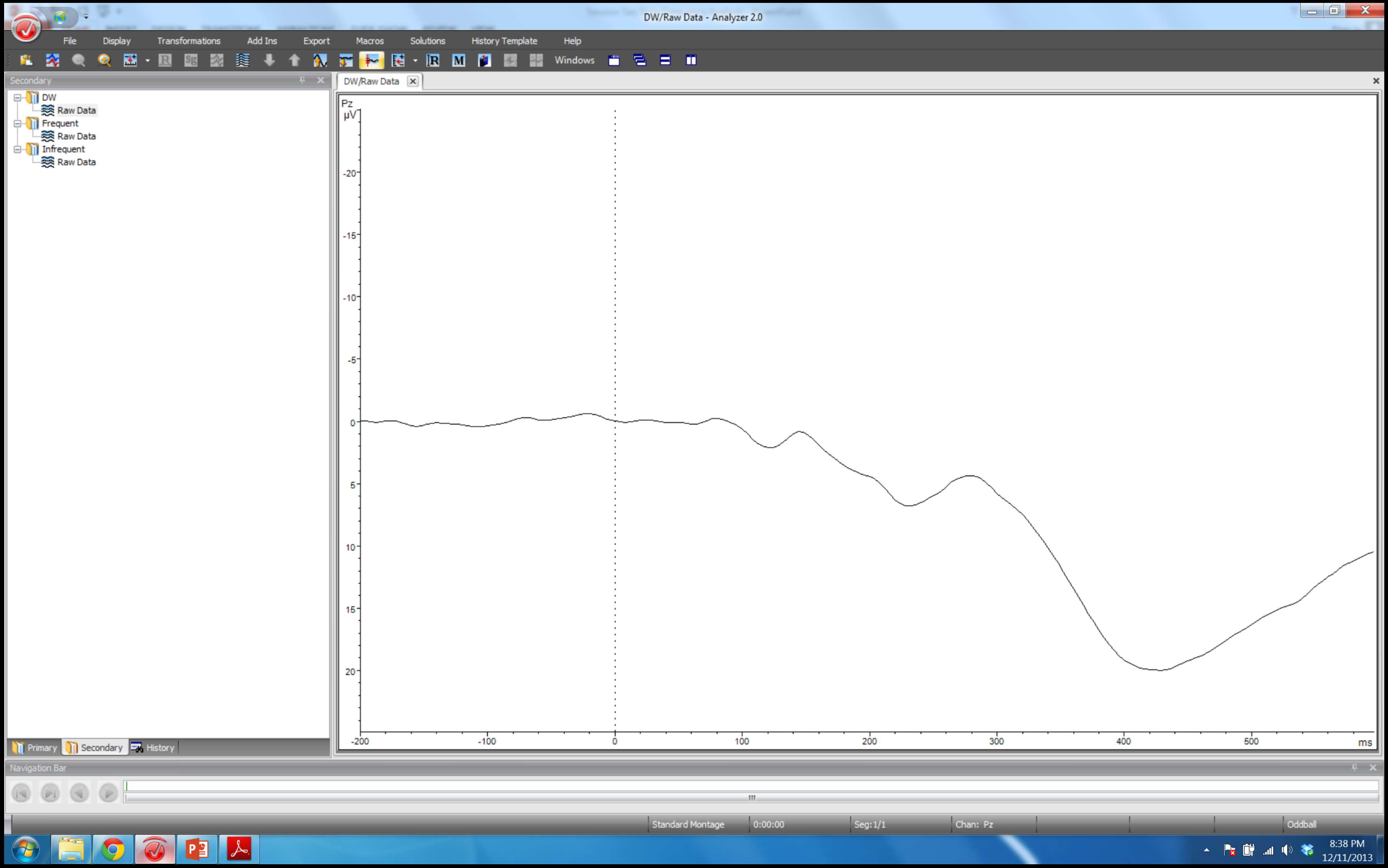
Time Frequency Analysis

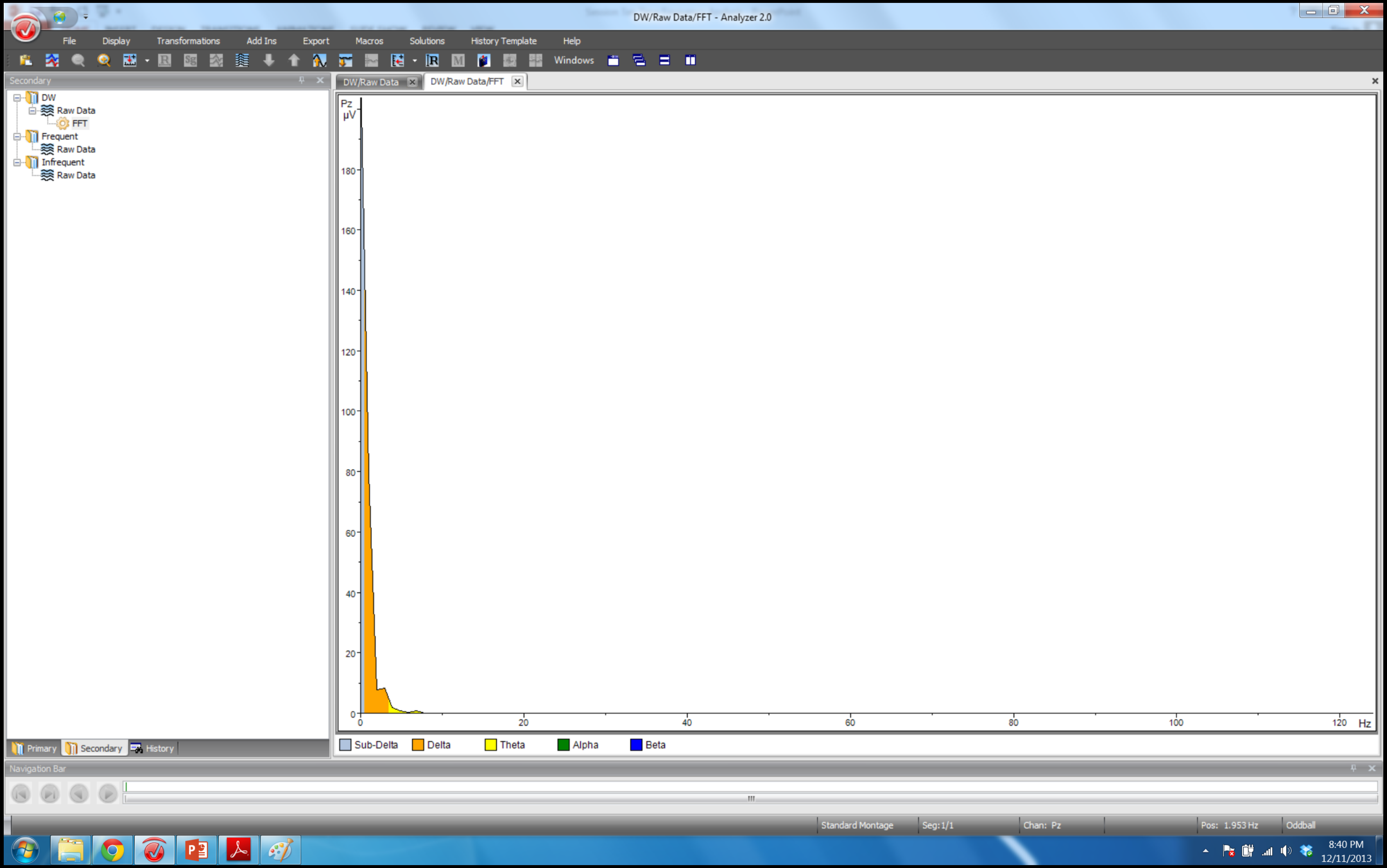

frequency
time

Time Domain

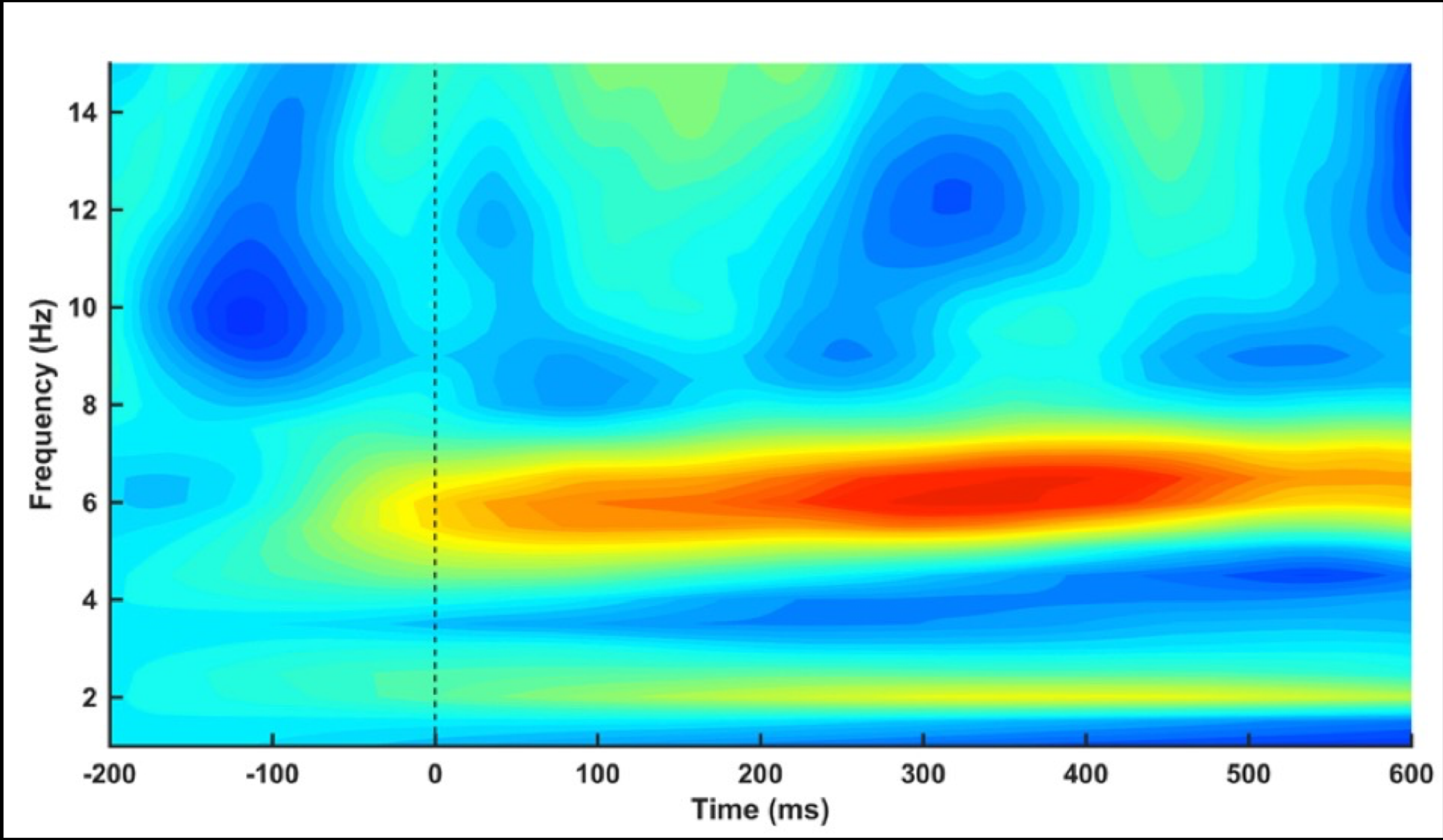


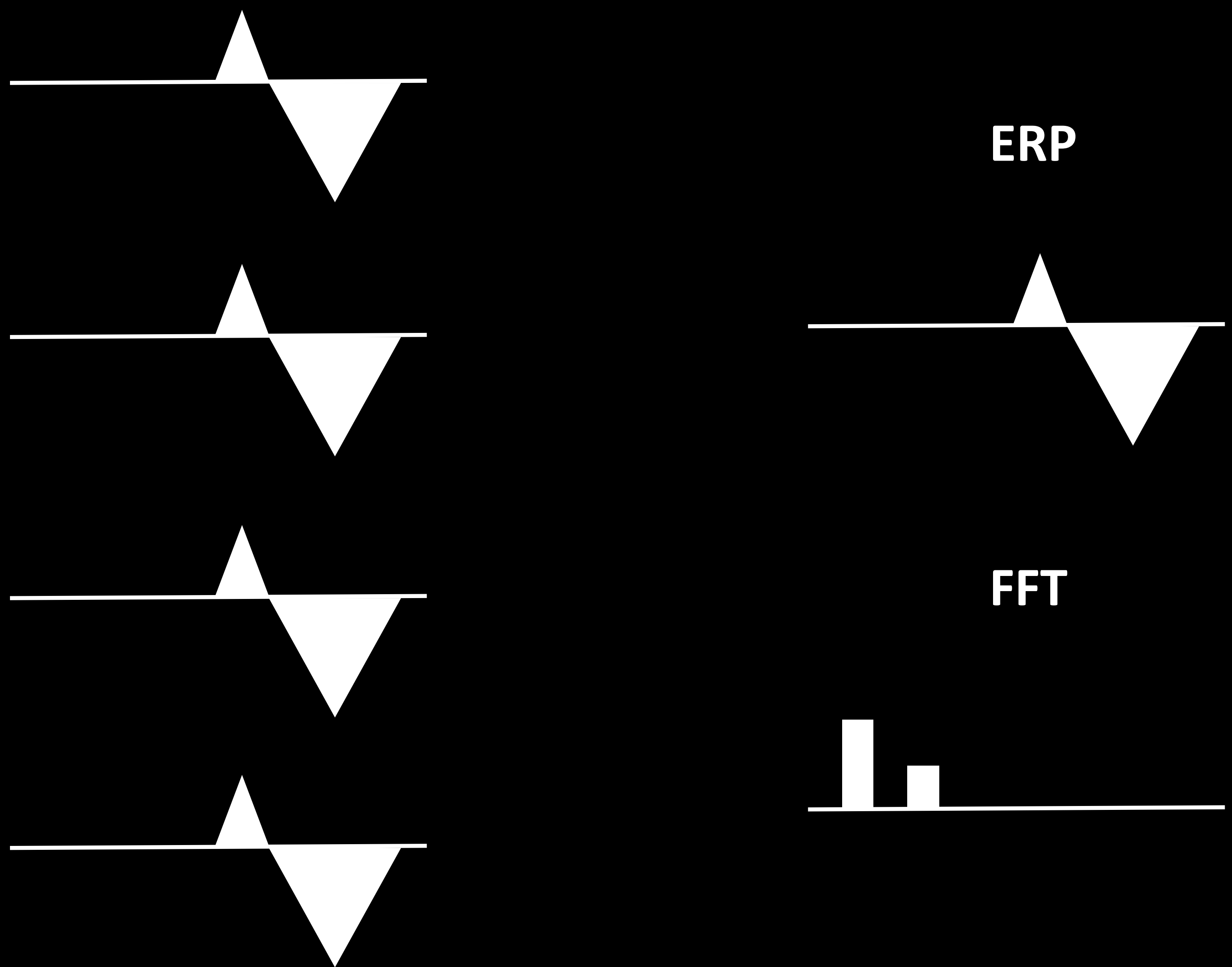
DW/Raw Data - Analyzer 2.0
File
Display
Transformations
Add Ins
Export
Macros
Solutions
History Template
Help
Windows
Secondary
DW
Raw Data
Frequent
Raw Data
Infrequent
Raw Data
DW/Raw Data
Pz
µV
-20
-15
-10
-5
0
5
10
15
20
-200
-100
0
100
200
300
400
500
ms
Primary
Secondary
History
Navigation Bar
Standard Montage
0:00:00
Seg: 1/1
Chan: Pz
Oddball
8:38 PM
12/11/2013

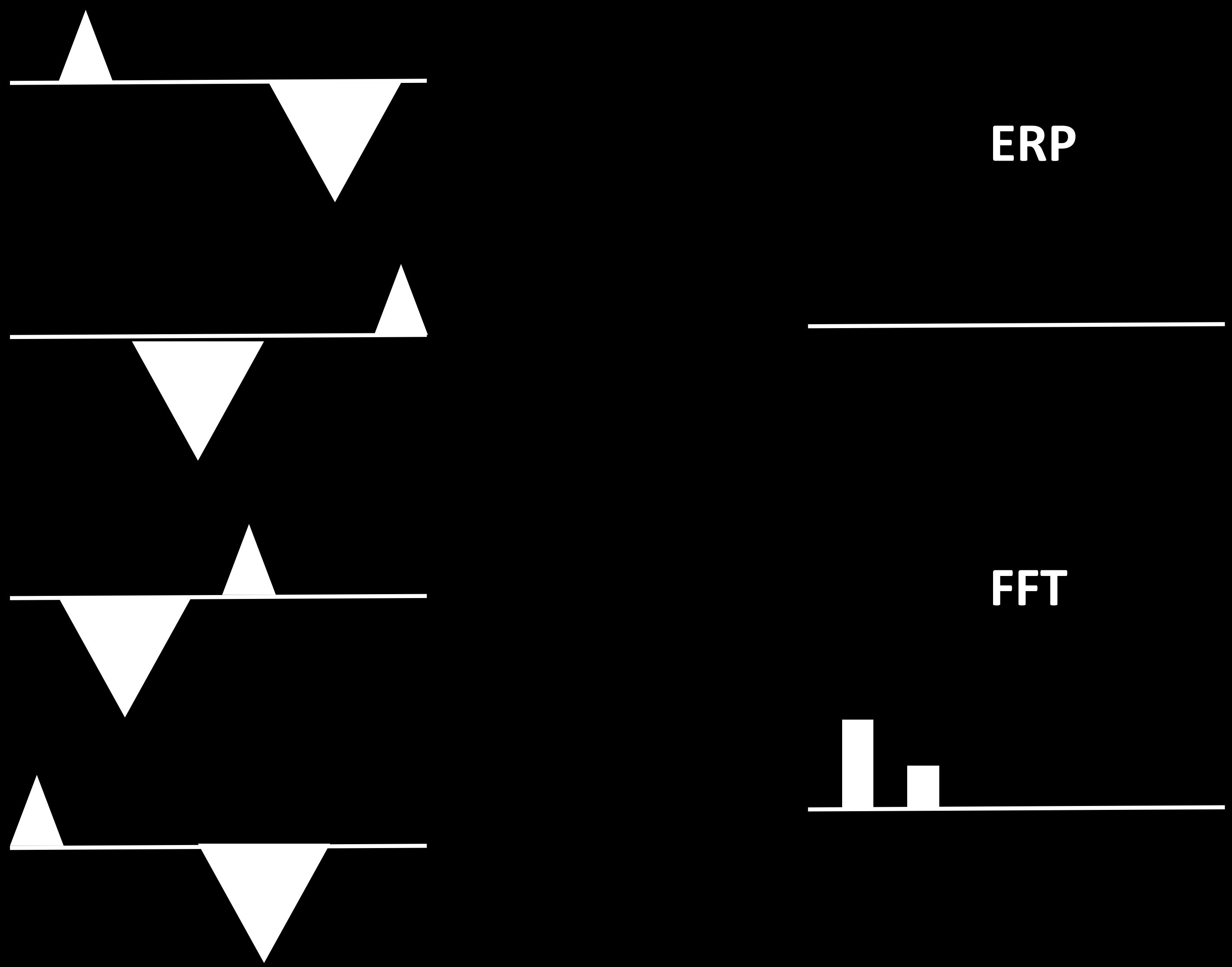
Frequency Domain



Time - Frequency Domain




ERP
FFT


ERP
FFT

So…

In general, this is the point. Frequency analysis allows examination of asynchronous (induced) activity that is not visible in an ERP analysis.

Time-Frequency analysis in principle captures both, but can also be harder to interpret.

Segmented Data = Evoked + Induced

Average Data = Evoked

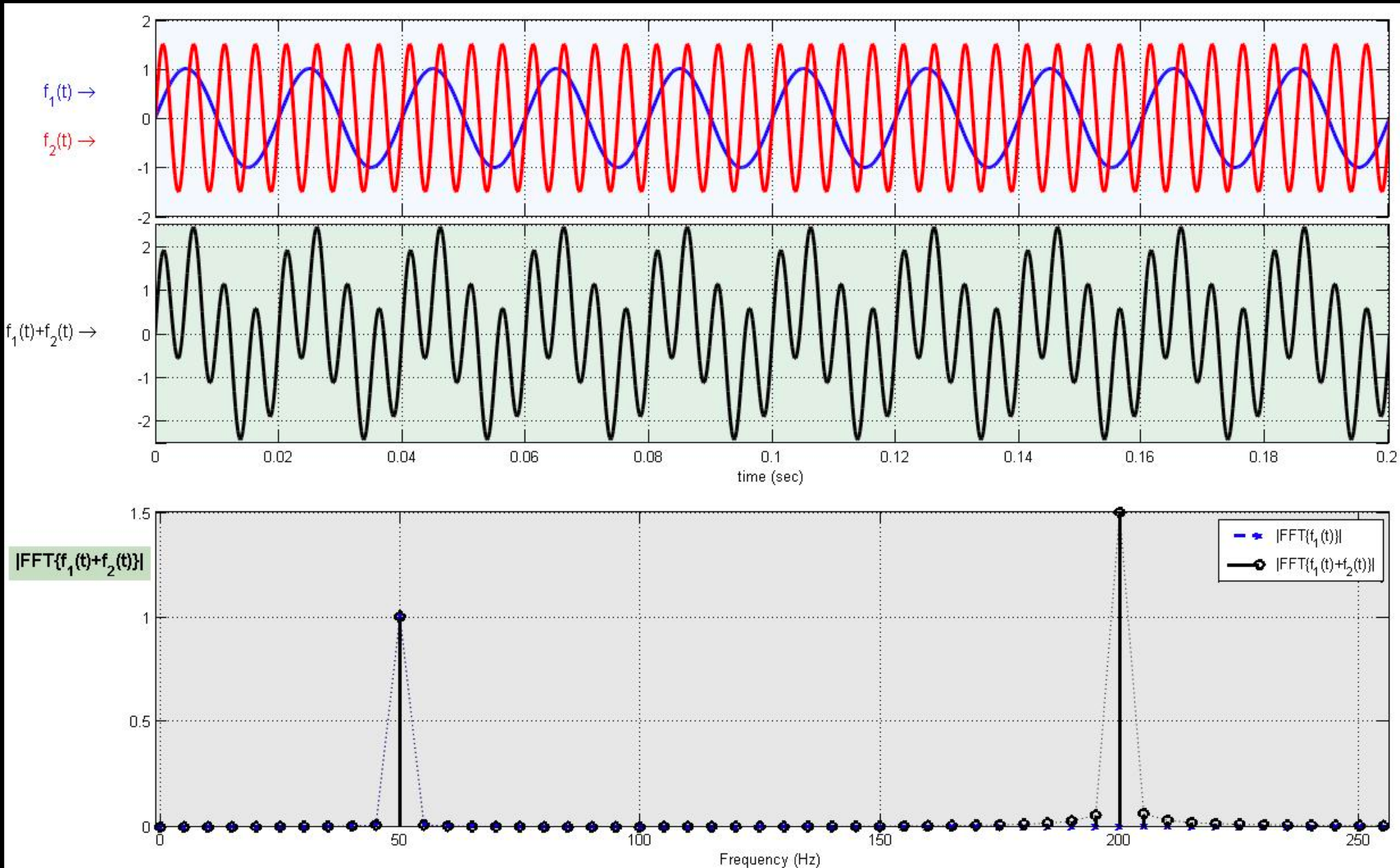
Segmented Data – Average Data = Induced

The Fourier Theorem

Recall…

Simply put, the Fourier theorem states that any waveform can be decomposed into a series of Sine waves.

As we noted previously, this is important in EEG research because it allows us to examine activity at different frequency bands.



$f_1(t) \rightarrow$
$f_2(t) \rightarrow$
$f_1(t)+f_2(t) \rightarrow$
time (sec)
|FFT{f_1(t)+f_2(t)}|
|FFT{f_1(t)}|
|FFT{f_1(t)+f_2(t)}|
Frequency (Hz)

Fast Fourier Transforms

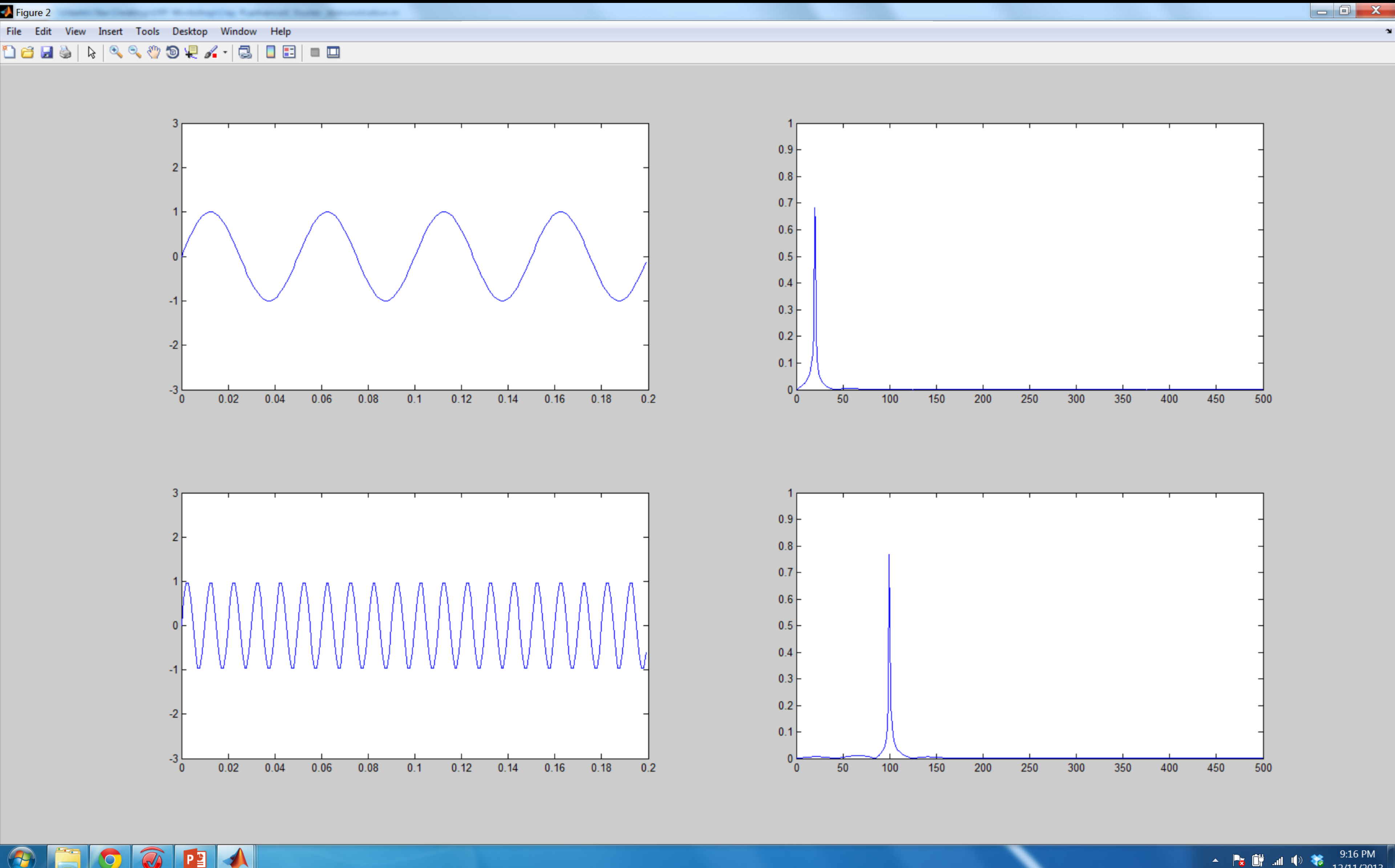
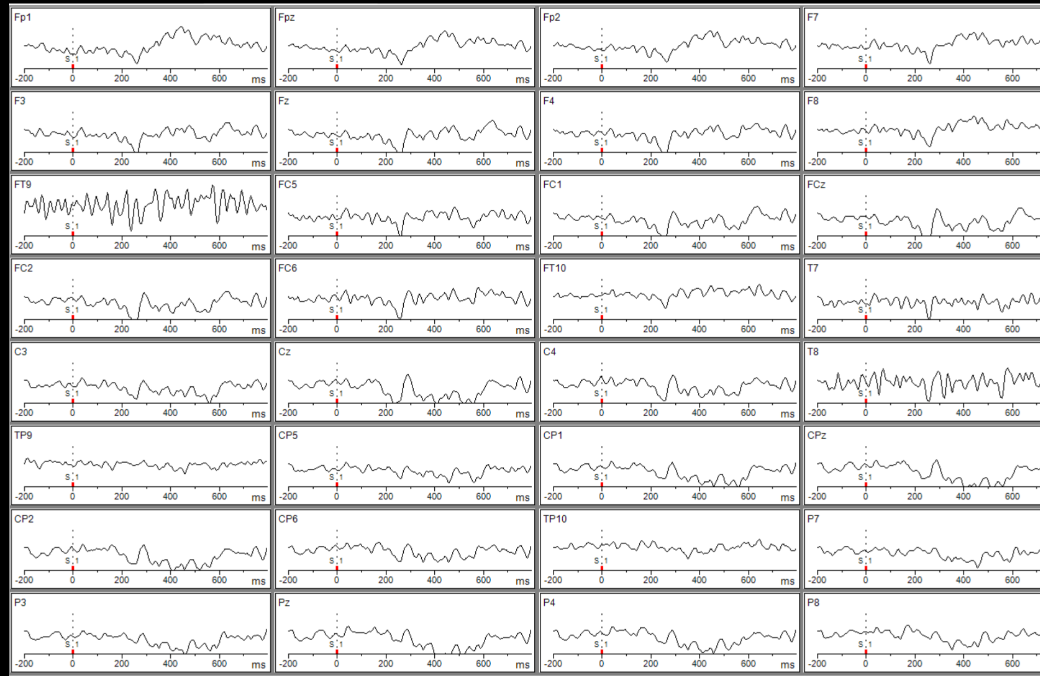
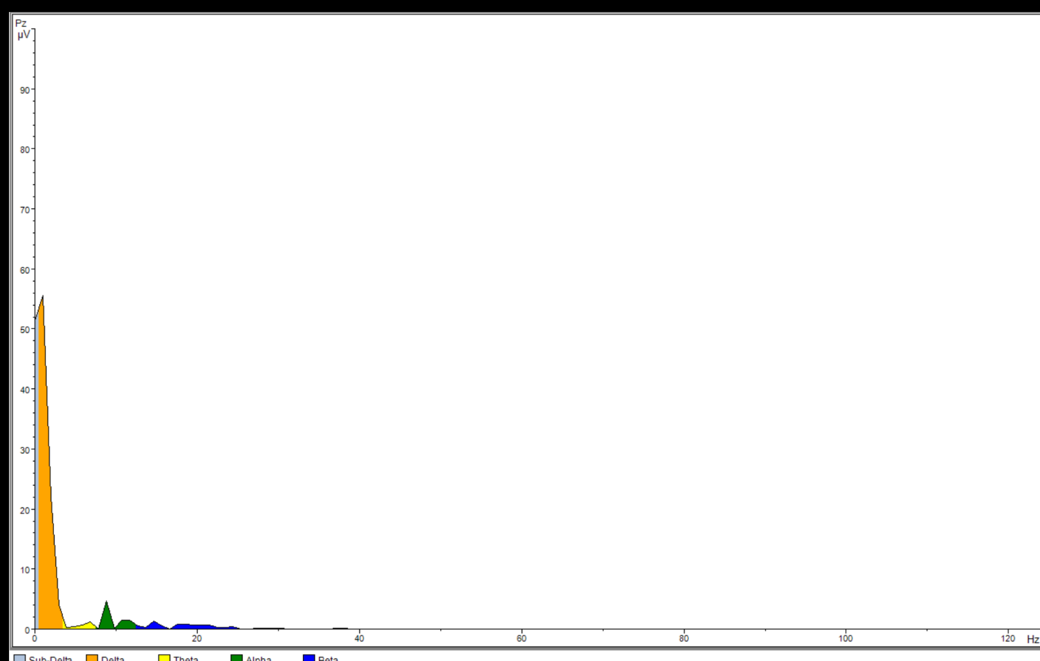


Figure 2
File Edit View Insert Tools Desktop Window Help
3 2 1 0 -1 -2 -3
0 0.02 0.04 0.06 0.08 0.1 0.12 0.14 0.16 0.18 0.2
1 0.9 0.8 0.7 0.6 0.5 0.4 0.3 0.2 0.1 0
0 50 100 150 200 250 300 350 400 450 500
3 2 1 0 -1 -2 -3
0 0.02 0.04 0.06 0.08 0.1 0.12 0.14 0.16 0.18 0.2
1 0.9 0.8 0.7 0.6 0.5 0.4 0.3 0.2 0.1 0
0 50 100 150 200 250 300 350 400 450 500
9:16 PM
12/11/2013


Fp1
Fpz
Fp2
F7
F3
Fz
F4
F8
FT9
FC5
FC1
FCz
FC2
FC6
FT10
T7
C3
Cz
C4
T8
TP9
CP5
CP1
CPz
CP2
CP6
TP10
P7
P3
Pz
P4
P8
-200
0
200
400
600
ms


Pz
µV
90
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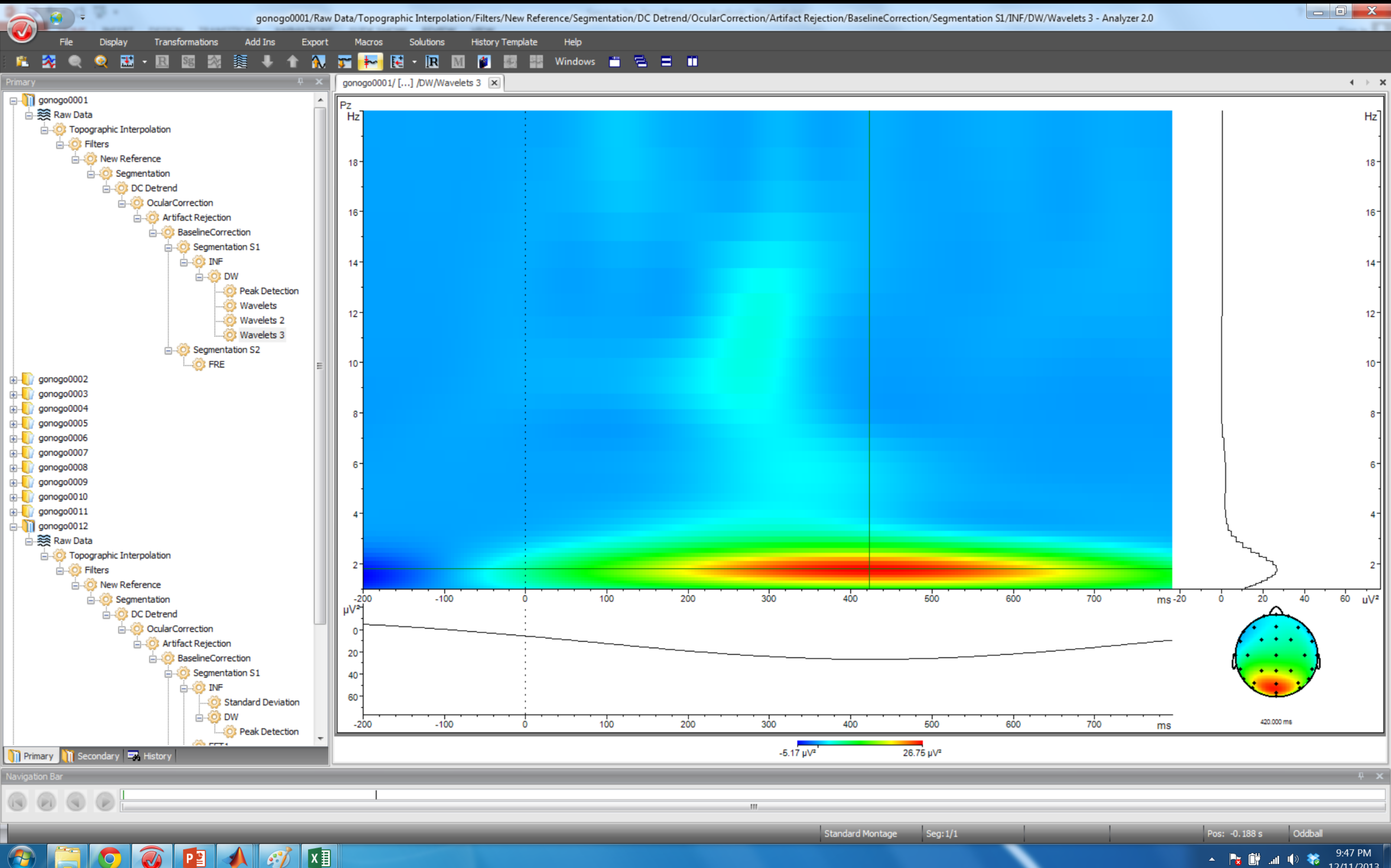
0
20
40
60
80
100
120
Hz
Sub-Delta
Delta
Theta
Alpha
Beta

So…

Essentially for each subject you can get a number that quantifies the amount of power in a given frequency band for each channel. Then, statistics can be done accordingly.

FFT Demo

Wavelet Analysis



gonogo0001/Raw Data/Topographic Interpolation/Filters/New Reference/Segmentation/DC Detrend/OcularCorrection/Artifact Rejection/BaselineCorrection/Segmentation S1/INF/DW/Wavelets 3 - Analyzer 2.0
File Display Transformations Add Ins Export Macros Solutions History Template Help
Windows
Primary
gonogo0001
Raw Data
Topographic Interpolation
Filters
New Reference
Segmentation
DC Detrend
OcularCorrection
Artifact Rejection
BaselineCorrection
Segmentation S1
INF
DW
Peak Detection
Wavelets
Wavelets 2
Wavelets 3
Segmentation S2
FRE
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gonogo0007
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gonogo0009
gonogo0010
gonogo0011
gonogo0012
Raw Data
Topographic Interpolation
Filters
New Reference
Segmentation
DC Detrend
OcularCorrection
Artifact Rejection
BaselineCorrection
Segmentation S1
INF
Standard Deviation
DW
Peak Detection
Primary Secondary History
gonogo0001/ [...] /DW/Wavelets 3
Pz
Hz
18 16 14 12 10 8 6 4 2
-200 -100 0 100 200 300 400 500 600 700 ms
-20 0 20 40 60 µV²
µV²
0 20 40 60
420.000 ms
-5.17 µV²
26.75 µV²
Navigation Bar
Standard Montage
Seg: 1/1
Pos: -0.188 s
Oddball
9:47 PM
12/11/2013

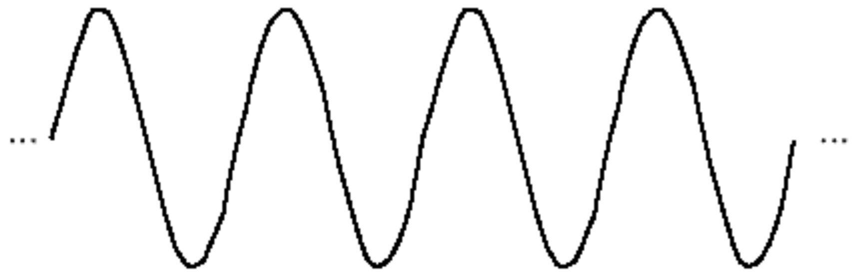
The idea…

Is relatively simple…

Instead of running a FFT over the entire window, a series of smaller overlapped FFTs are run over the time window thus allowing a map of frequency against time.

What is Wavelet Analysis ?

- And…what is a wavelet…?



Sine Wave

Wavelet (db10)

- A wavelet is a waveform of effectively limited duration that has an average value of zero.

The Continuous Wavelet Transform (CWT)

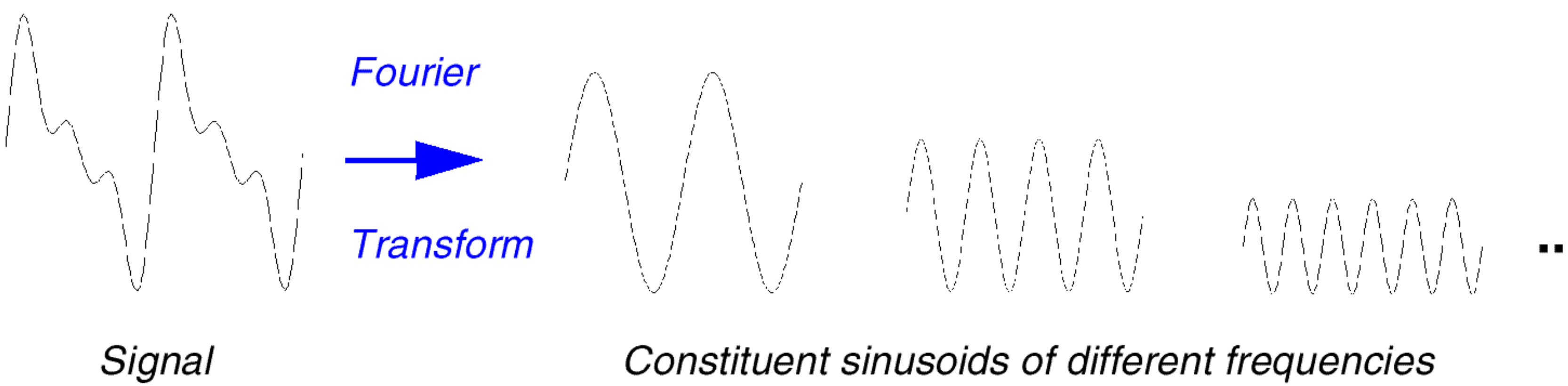
- A mathematical representation of the Fourier transform:

$$F(w) = \int_{-\infty}^{\infty} f(t) e^{-iwt} dt$$

- Meaning: the sum over all time of the signal *f(t)* multiplied by a complex exponential, and the result is the Fourier coefficients F(?) .

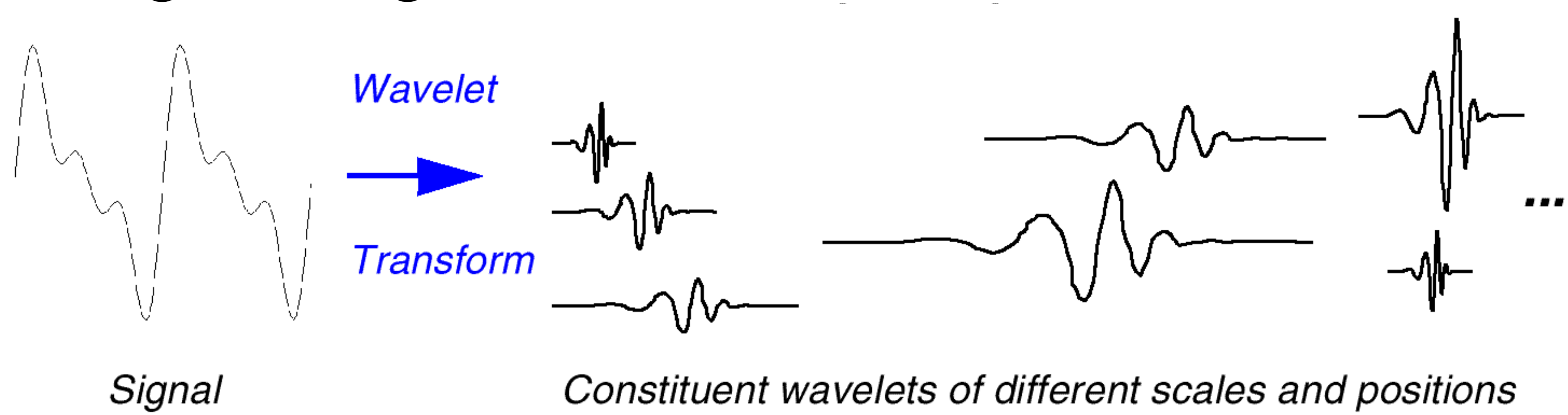
Wavelet Transform (Cont'd)

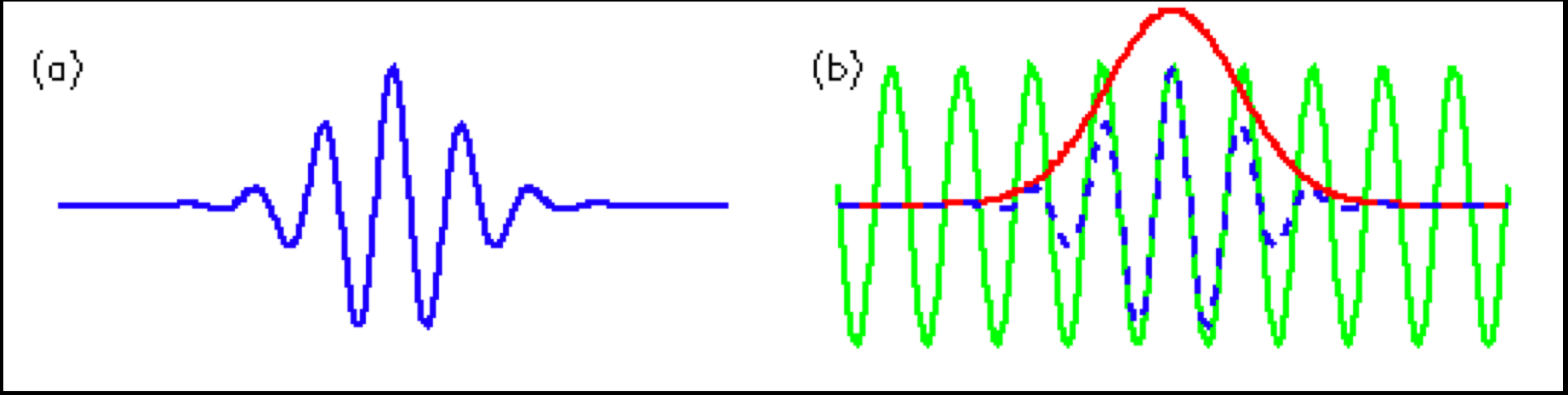
- Those coefficients, when multiplied by a sinusoid of appropriate frequency ⍰, yield the constituent sinusoidal component of the original signal:



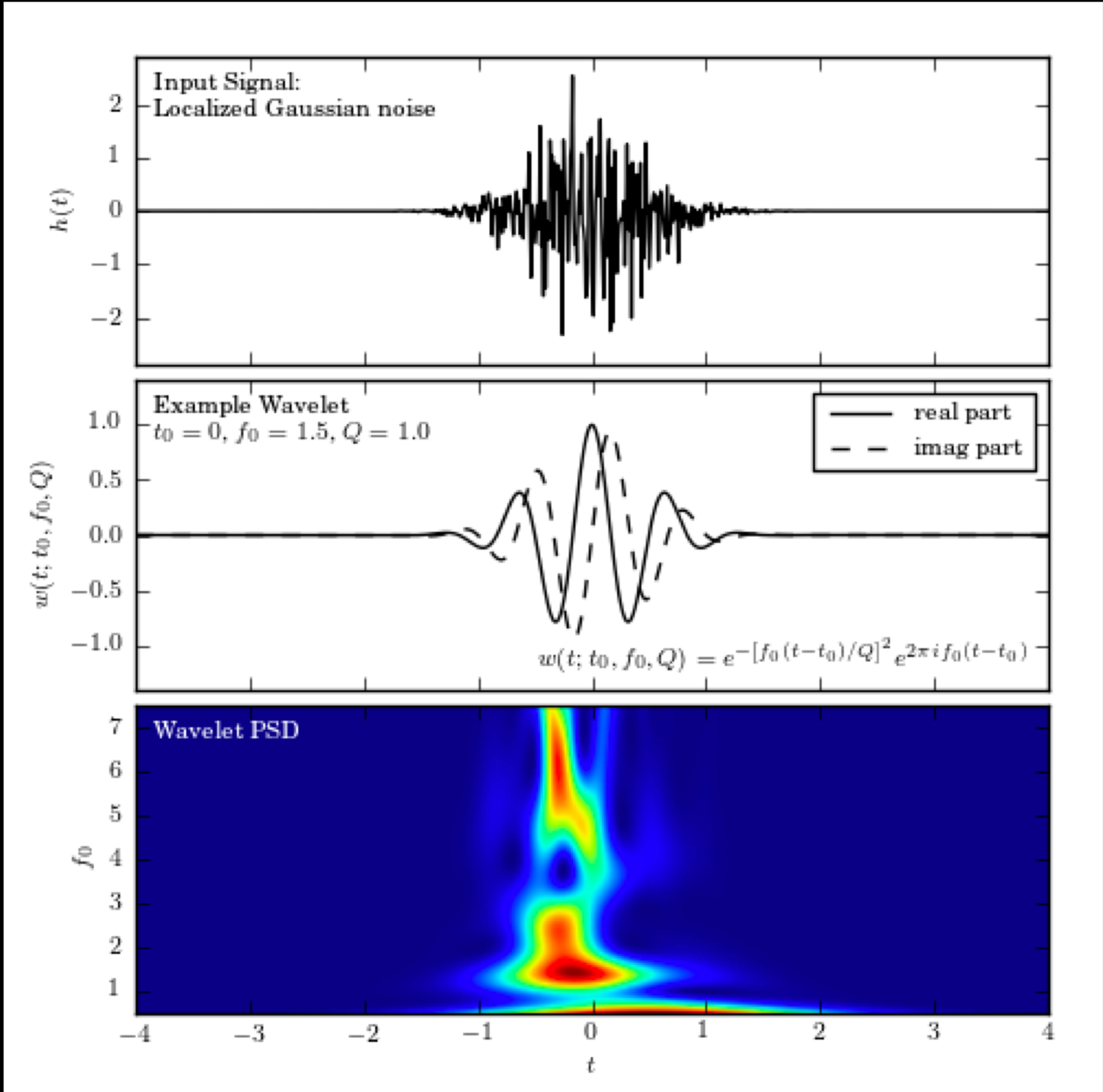
Wavelet Transform

- And the result of the CWT are Wavelet coefficients .
- Multiplying each coefficient by the appropriately scaled and shifted wavelet yields the constituent wavelet of the original signal:





(a)
(b)



Input Signal:
Localized Gaussian noise
$h(t)$
Example Wavelet
$t_0 = 0$, $f_0 = 1.5$, $Q = 1.0$
real part
imag part
$w(t; t_0, f_0, Q)$
$w(t; t_0, f_0, Q) = e^{-[f_0(t-t_0)/Q]^2} e^{2\pi i f_0 (t-t_0)}$
Wavelet PSD
f_0
t

Wavelet function

$$\Psi_{a,b}(x) = \frac{1}{\sqrt{a}} \Psi\left(\frac{x-b}{a}\right)$$

- b – shift coefficient
- a – scale coefficient

$$\Psi_{a,b_x,b_y}(x,y) = \frac{1}{|a|} \Psi\left(\frac{x-b_x}{a}, \frac{y-b_y}{a}\right)$$

- 2D function

CWT

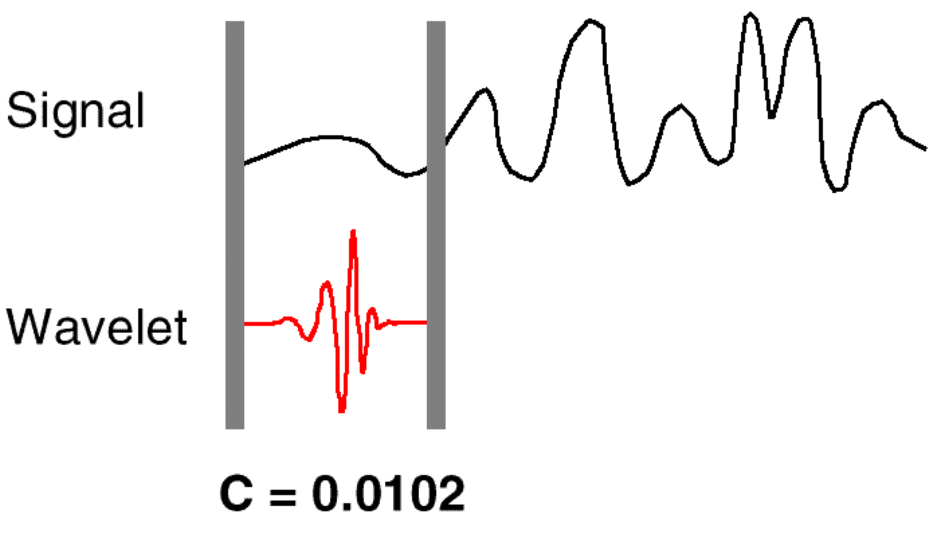
- Reminder: The *CWT* Is the sum over all time of the signal, multiplied by scaled and shifted versions of the wavelet function

Step 1:
Take a Wavelet and compare
it to a section at the start
of the original signal

CWT

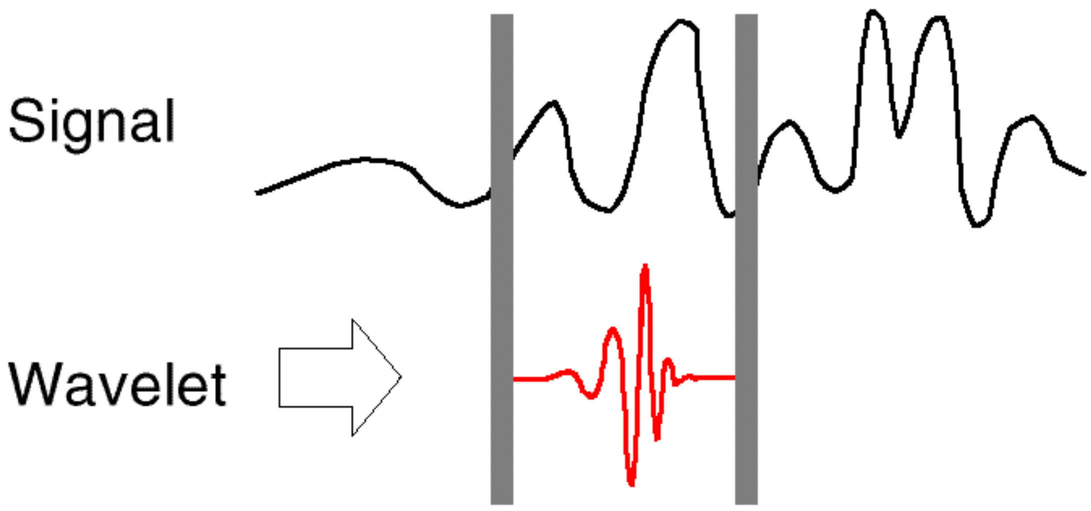
<u>Step 2:</u>

Calculate a number, C, that represents how closely correlated the wavelet is with this section of the signal. The higher C is, the more the similarity.



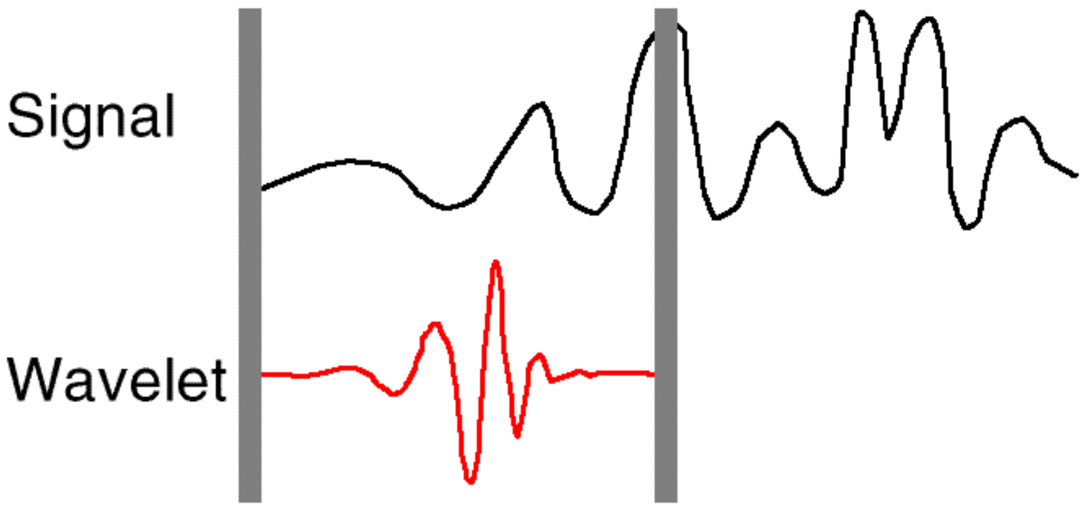
CWT

- <u>Step 3:</u> Shift the wavelet to the right and repeat steps 1-2 until you've covered the whole signal



CWT

- Step 4: Scale (stretch) the wavelet and repeat steps 1-3



C = 0.2247

Wavelets

Some argue this is the most accurate representation of the data because the activity reflected in the wavelet analysis is:

Evoked + Induced

The only real criticism is that for this to be a true statement there can be no edge artifacts – the wavelet window has to capture all of the data.

Wavelets

A wavelet analysis can be run on segmented (evoked + induced) or average (evoked) data.

It is important to note if you only run wavelets on average data, you are going to get the exact same representation as the time domain, but instead in the frequency domain.

As such, wavelets should be run on segmented data and then averaged across participants.

Note, you lose nothing in this average process – it is a very different concept than the time domain analog.

Wavelets and Preprocessing

1. Topographic Interpolation

   wavelets are susceptible to bad channels

2. Filtering

   you may wish to use a much wider bandwidth here, the issues that plague ERPs are not nearly as important

3. Re-referencing

   There is debate about this – you need some form of reference but the answer is not as clear

Wavelets and Preprocessing

4. Segmentation

   Wavelets do not work well on continuous data

   Note, a FFT can be run on a reasonable length of artifact free continuous data

5. Ocular Correction

   Should be done

6. DC Detrend

   Not necessary

7. Artifact Rejection

   Should be done

Wavelets and Preprocessing

8. Baseline Correction

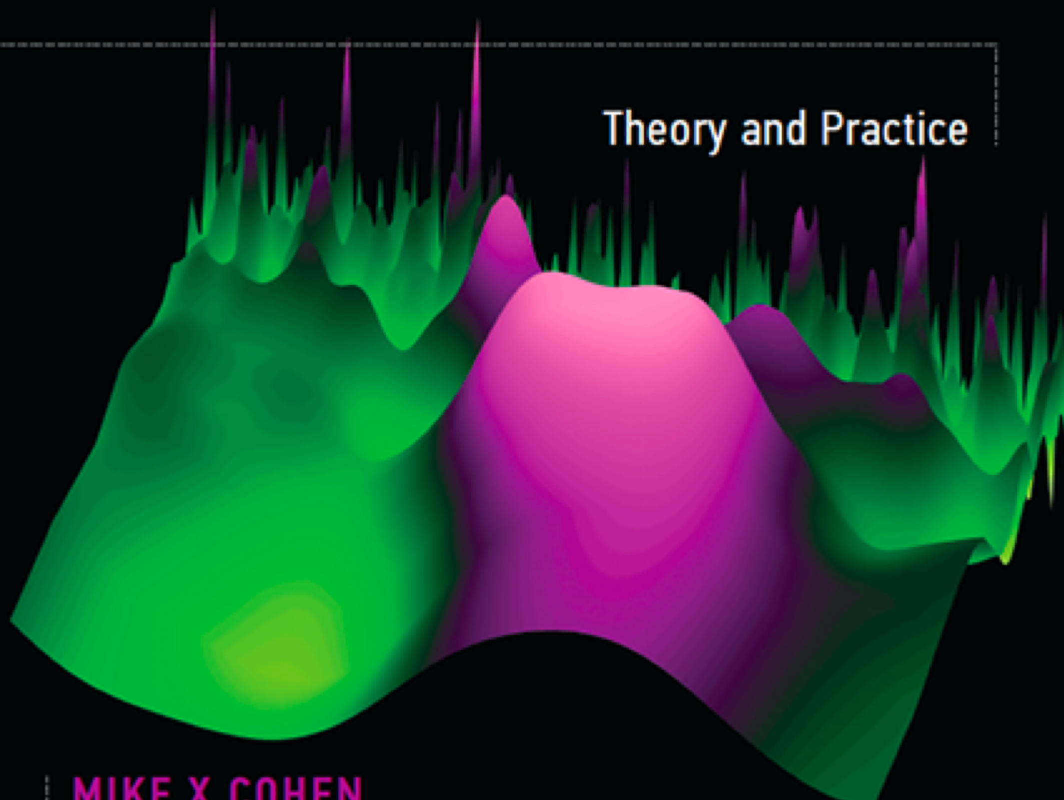
   Is built into the procedure but needs to be done on the time data as well. Also, typically a further out baseline is needed, say -300 to -100.

9. Averaging

Finally, there are a lot of parameters when running a wavelet analysis. There is no clear statement on the correct ones to use.

ANALYZING NEURAL TIME SERIES DATA

Theory and Practice



MIKE X COHEN

Wavelet Demo