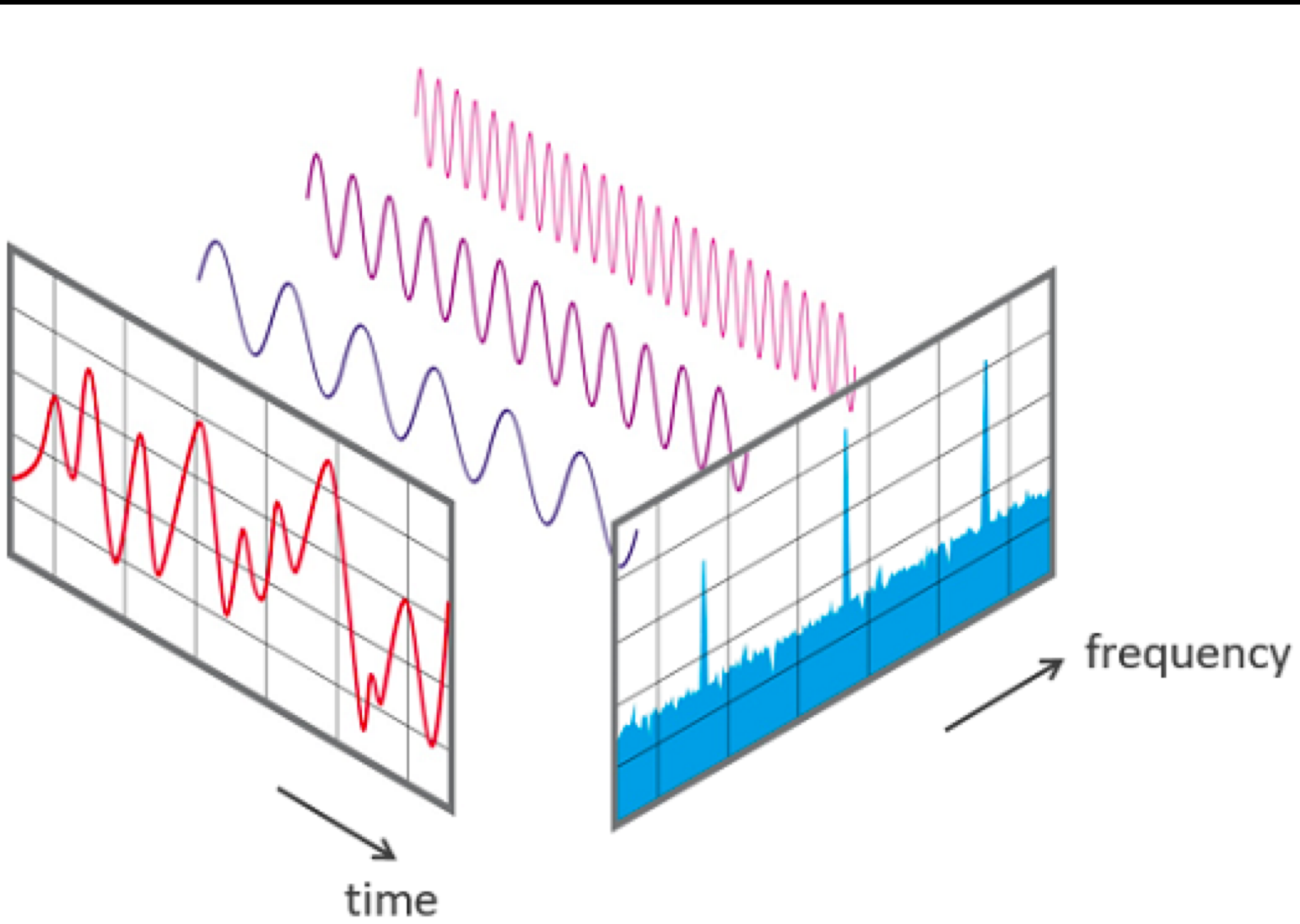
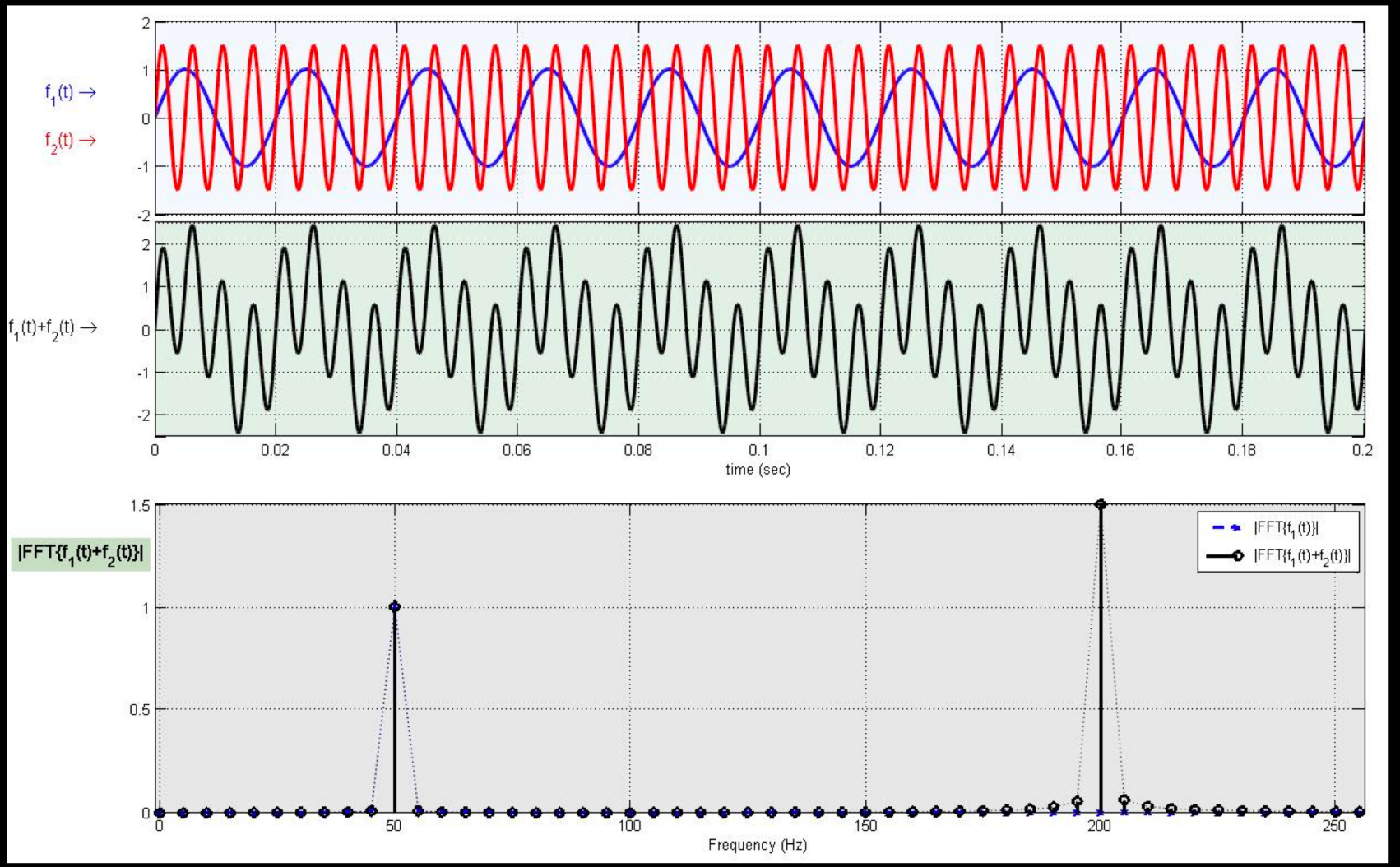
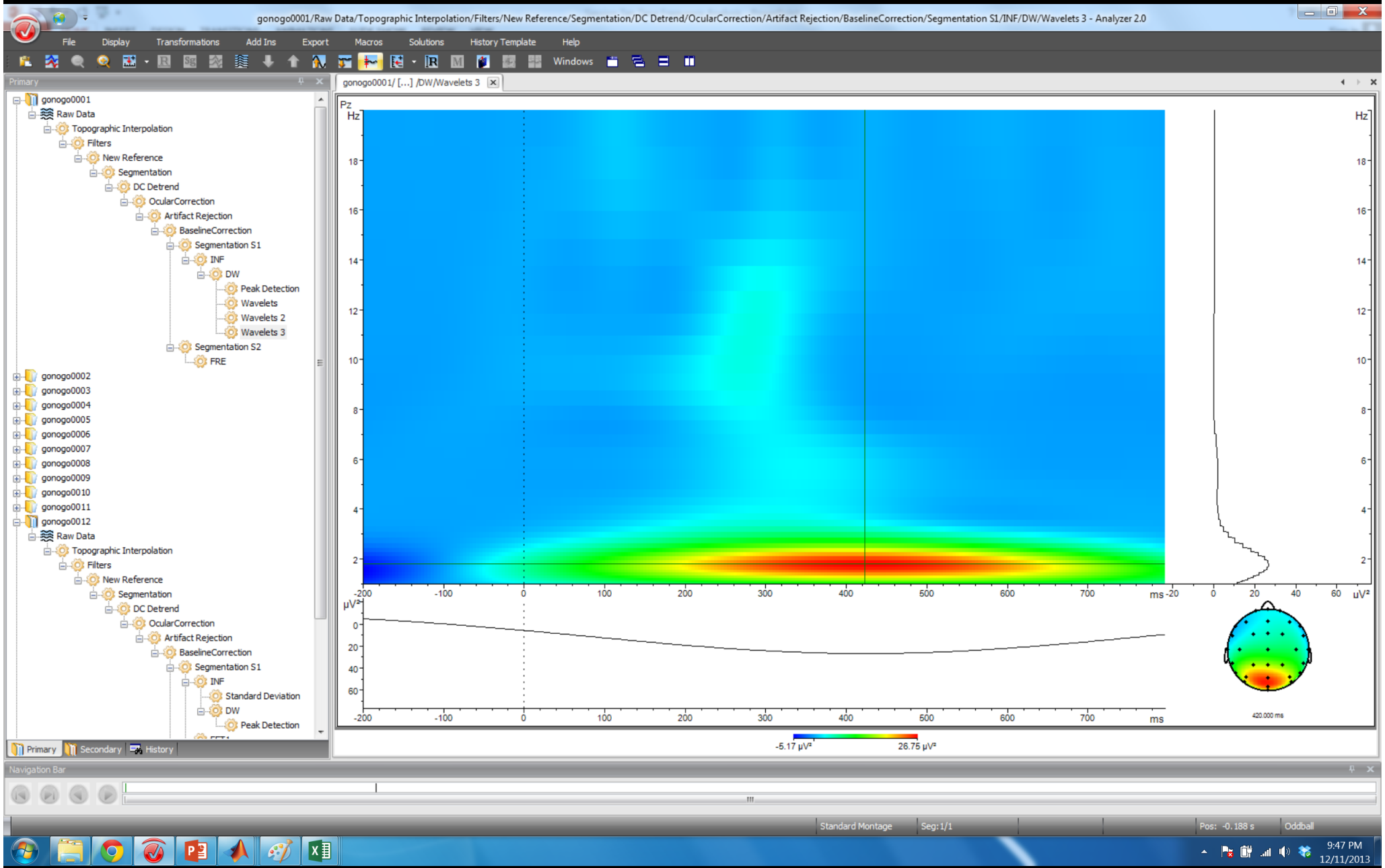
# Wavelet Analysis


frequency
time


f1(t) →
f2(t) →
f1(t)+f2(t) →
time (sec)
|FFT{f1(t)+f2(t)}|
|FFT{f1(t)}|
|FFT{f1(t)+f2(t)}|
Frequency (Hz)


gonogo0001/Raw Data/Topographic Interpolation/Filters/New Reference/Segmentation/DC Detrend/OcularCorrection/Artifact Rejection/BaselineCorrection/Segmentation S1/INF/DW/Wavelets 3 - Analyzer 2.0
File
Display
Transformations
Add Ins
Export
Macros
Solutions
History Template
Help
Windows
Primary
gonogo0001
Raw Data
Topographic Interpolation
Filters
New Reference
Segmentation
DC Detrend
OcularCorrection
Artifact Rejection
BaselineCorrection
Segmentation S1
INF
DW
Peak Detection
Wavelets
Wavelets 2
Wavelets 3
Segmentation S2
FRE
gonogo0002
gonogo0003
gonogo0004
gonogo0005
gonogo0006
gonogo0007
gonogo0008
gonogo0009
gonogo0010
gonogo0011
gonogo0012
Standard Deviation
gonogo0001/ [...] /DW/Wavelets 3
Pz
Hz
ms
μV²
-5.17 μV²
26.75 μV²
420.000 ms
Primary
Secondary
History
Navigation Bar
Standard Montage
Seg: 1/1
Pos: -0.188 s
Oddball
9:47 PM
12/11/2013

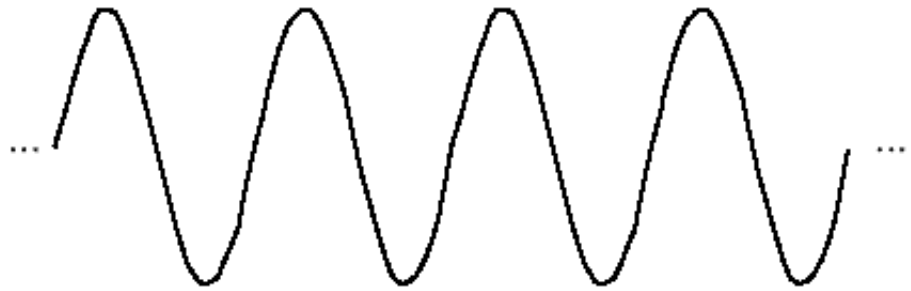
# The idea…

Is relatively simple…

Instead of running a FFT over the entire window, a series of smaller overlapped FFTs are run over the time window thus allowing a map of frequency against time (not quite).

# What is Wavelet Analysis ?

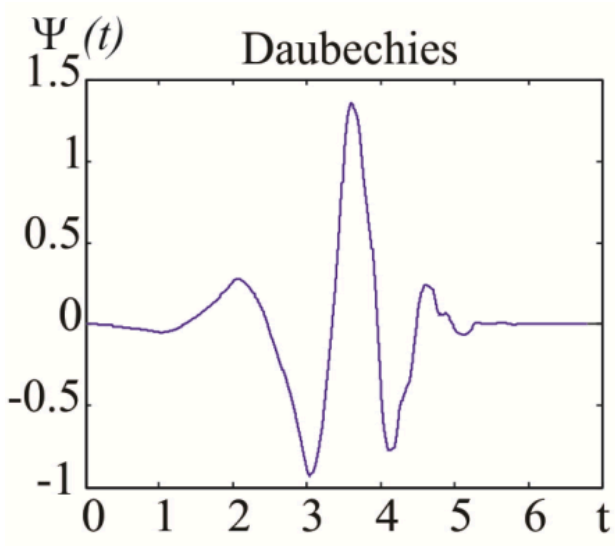
- And…what is a wavelet…?

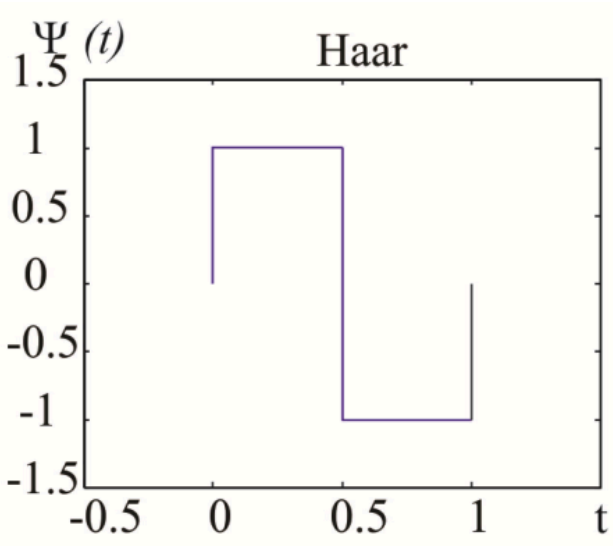


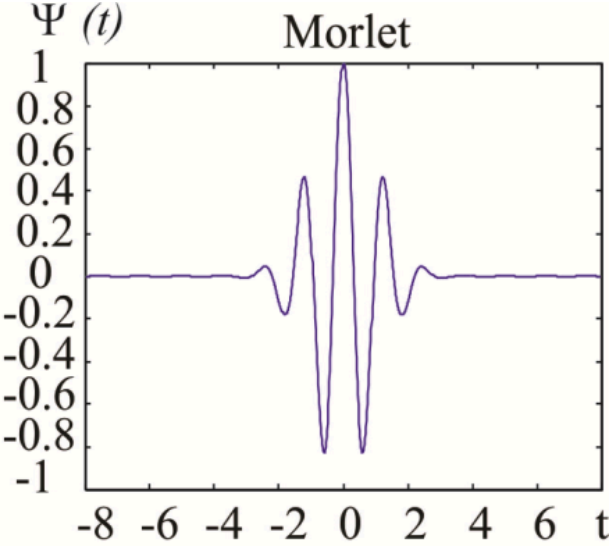
Sine Wave

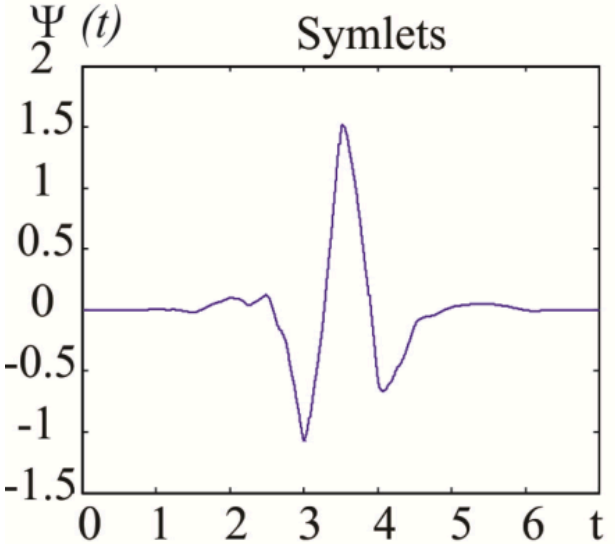
Wavelet (db10)

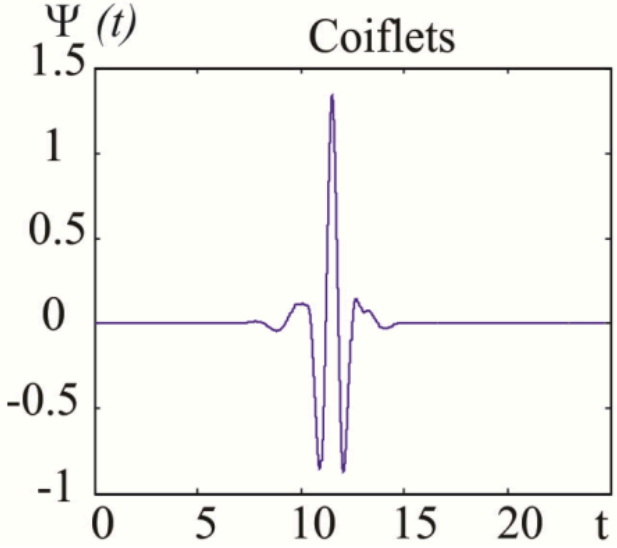
- A wavelet is a waveform of effectively limited duration that has an average value of zero.

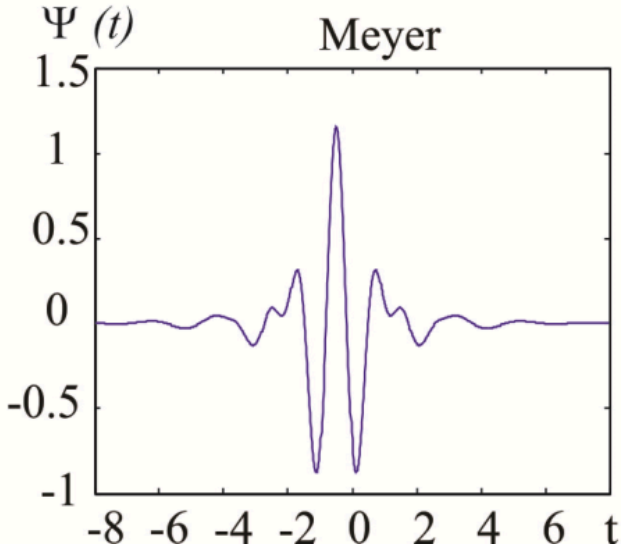

Ψ (t)
Daubechies
1.5
1
0.5
0
-0.5
-1
0 1 2 3 4 5 6 t

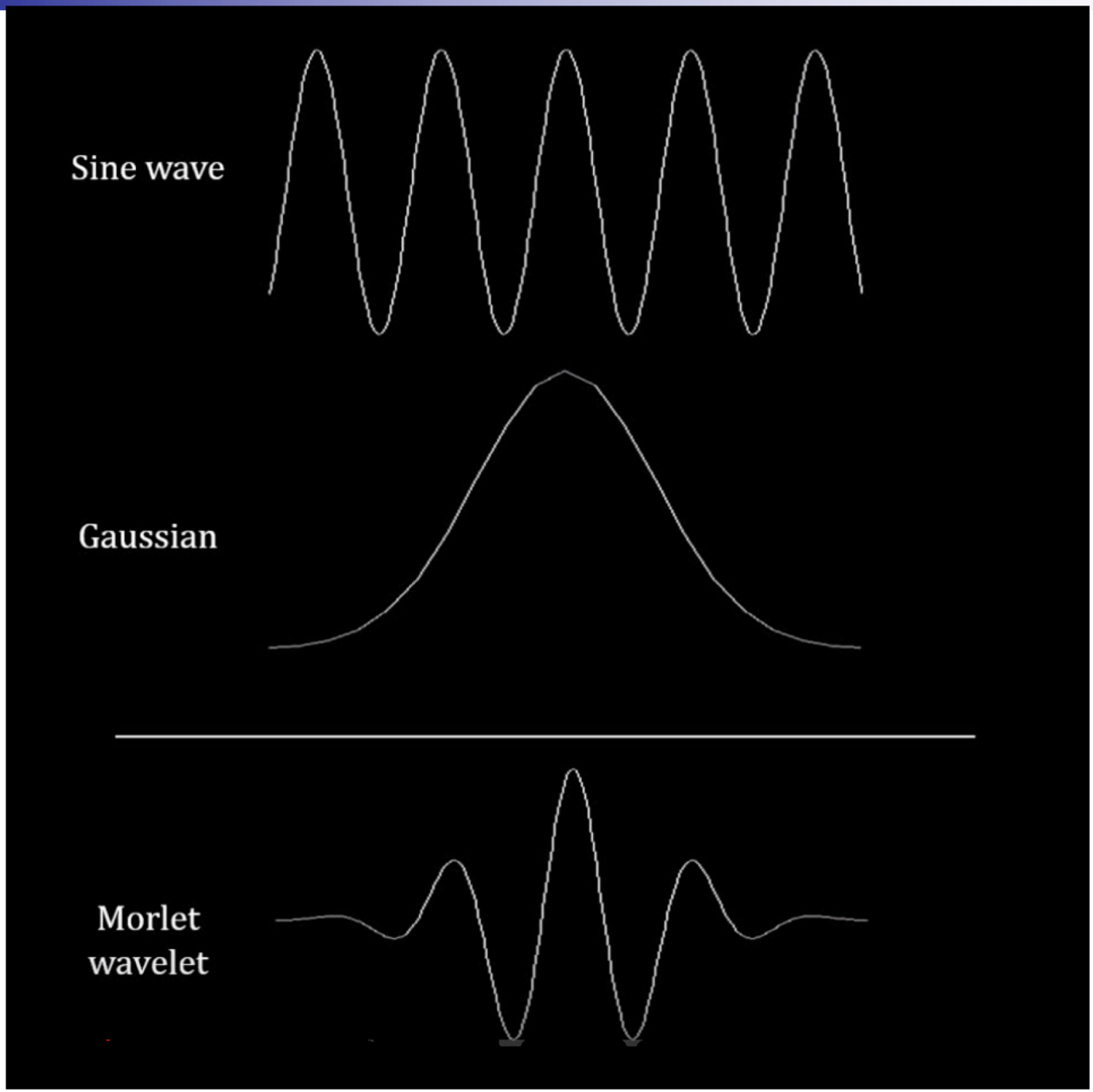

Ψ (t)
Haar
1.5
1
0.5
0
-0.5
-1
-1.5
-0.5 0 0.5 1 t

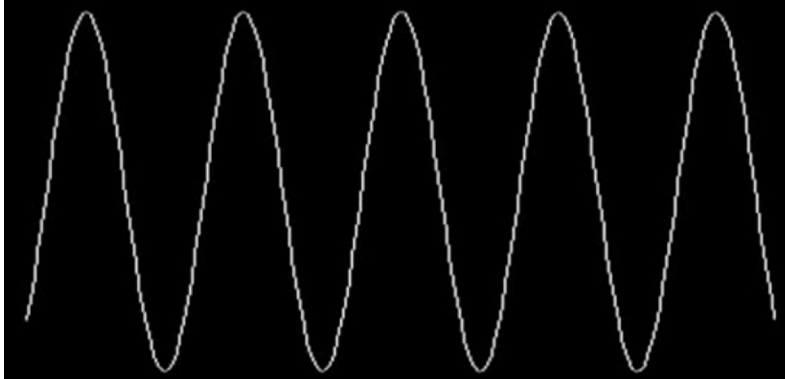

Ψ (t)
Morlet
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-8 -6 -4 -2 0 2 4 6 t


Ψ (t)
Symlets
2
1.5
1
0.5
0
-0.5
-1
-1.5
0 1 2 3 4 5 6 t

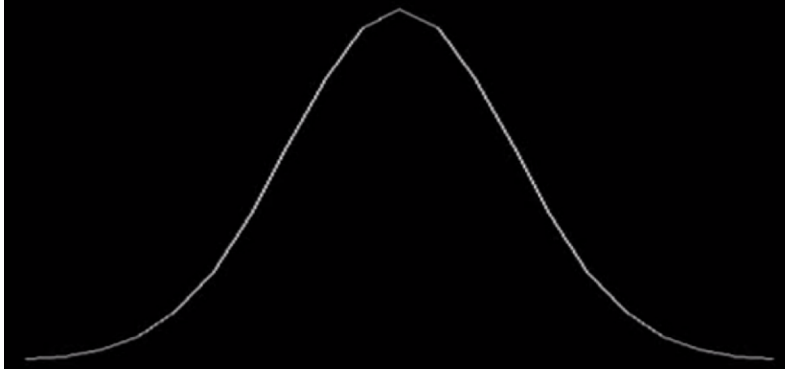

Ψ (t)
Coiflets
1.5
1
0.5
0
-0.5
-1
0 5 10 15 20 t


Ψ (t)
Meyer
1.5
1
0.5
0
-0.5
-1
-8 -6 -4 -2 0 2 4 6 t

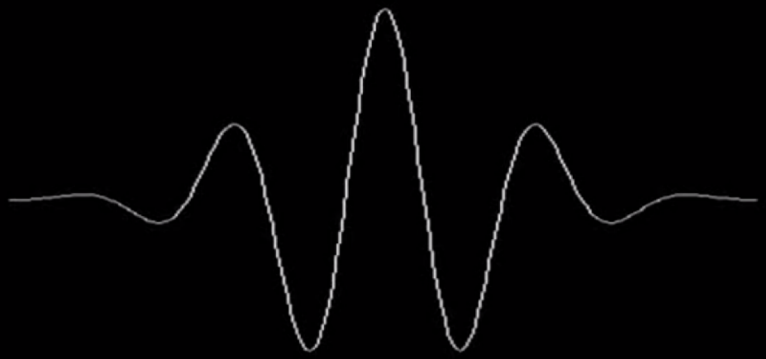

Sine wave
Gaussian
Morlet
wavelet



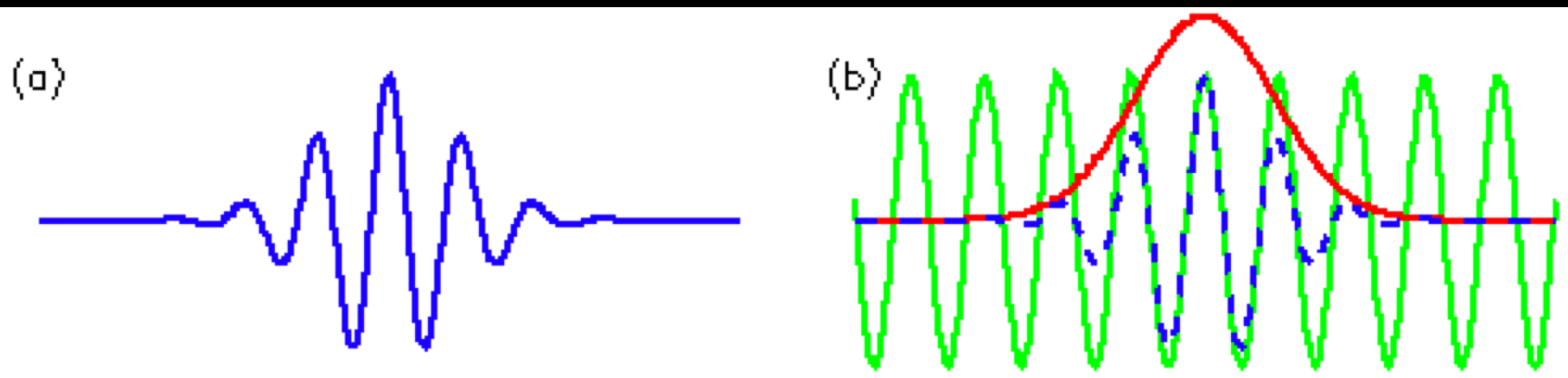
```
> sin(2*pi*freq*time);
```



```
> exp(-time^2/(2*s^2));
```



```
> sin(2*pi*freq*time).*
  exp(-time^2/(2*s2));
```




A wavelet is the convolution of a
Sine Wave and a Gaussian

# **The** Continuous Wavelet Transform (CWT)

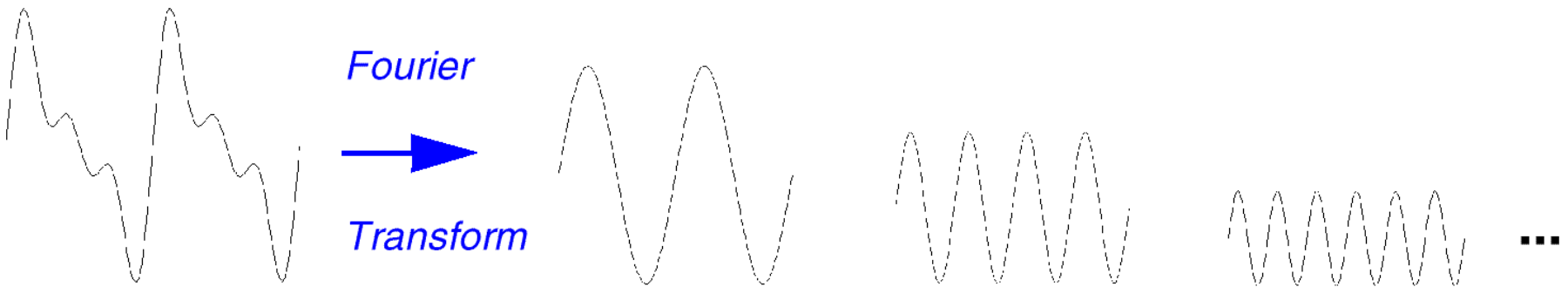
- A mathematical representation of the Fourier transform:

$$F(w) = \int_{-\infty}^{\infty} f(t) e^{-iwt} dt$$

- Meaning: the sum over all time of the signal *f(t)* multiplied by a complex exponential, and the result is the Fourier coefficients F(□) .

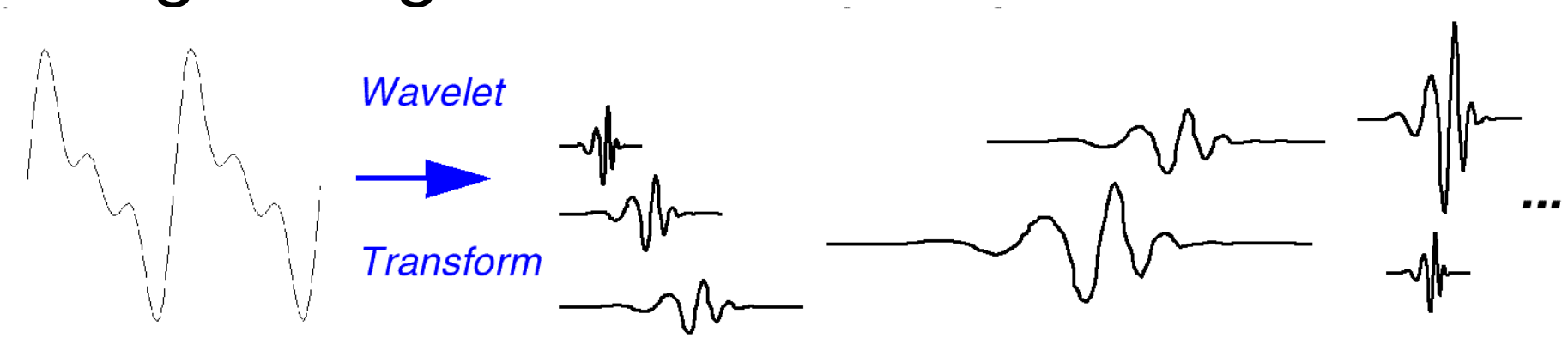
# Wavelet Transform (Cont'd)

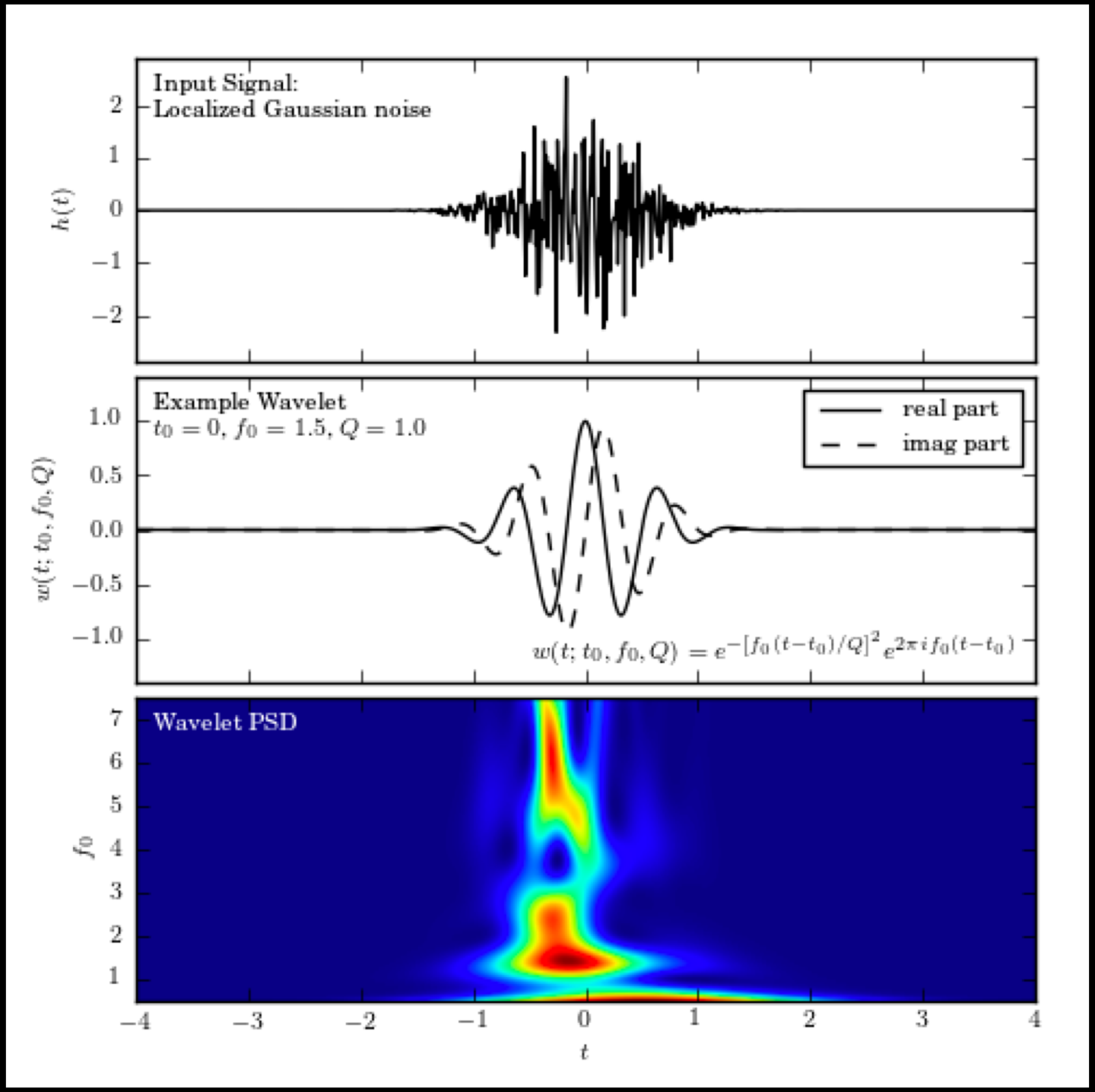
- Those coefficients, when multiplied by a sinusoid of appropriate frequency ?, yield the constituent sinusoidal component of the original signal:



# Wavelet Transform

- And the result of the CWT are Wavelet coefficients .
- Multiplying each coefficient by the appropriately scaled and shifted wavelet yields the constituent wavelet of the original signal:




Input Signal:
Localized Gaussian noise
h(t)
Example Wavelet
$t_0 = 0$, $f_0 = 1.5$, $Q = 1.0$
real part
imag part
$w(t; t_0, f_0, Q)$
$w(t; t_0, f_0, Q) = e^{-[f_0(t-t_0)/Q]^2} e^{2\pi i f_0 (t-t_0)}$
Wavelet PSD
$f_0$
$t$

# Wavelet function

$$\Psi_{a,b}(x) = \frac{1}{\sqrt{a}} \Psi\left(\frac{x-b}{a}\right)$$

- b – shift coefficient
- a – scale coefficient

$$\Psi_{a,b_x,b_y}(x,y) = \frac{1}{|a|} \Psi\left(\frac{x-b_x}{a}, \frac{y-b_y}{a}\right)$$

- 2D function

# CWT

- Reminder: The *CWT* Is the sum over all time of the signal, multiplied by scaled and shifted versions of the wavelet function

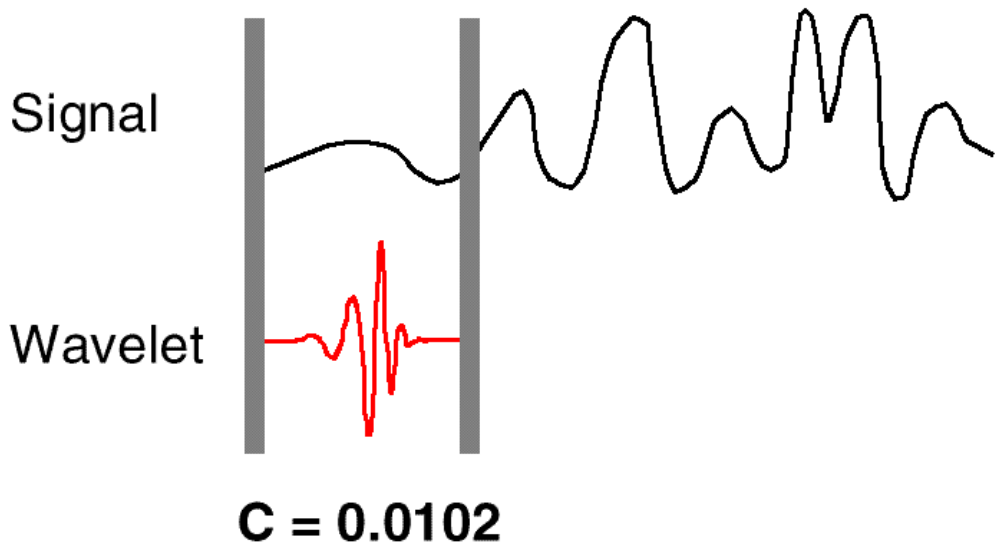
Step 1:

Take a Wavelet and compare it to a section at the start of the original signal

# CWT

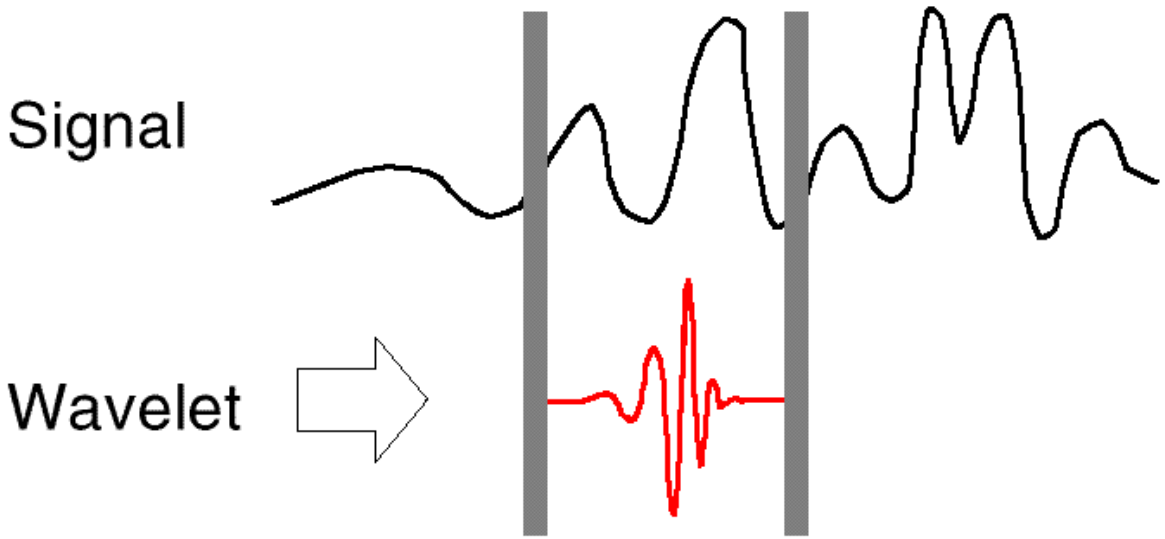
Step 2:

Calculate a number, C, that represents how closely correlated the wavelet is with this section of the signal. The higher C is, the more the similarity.



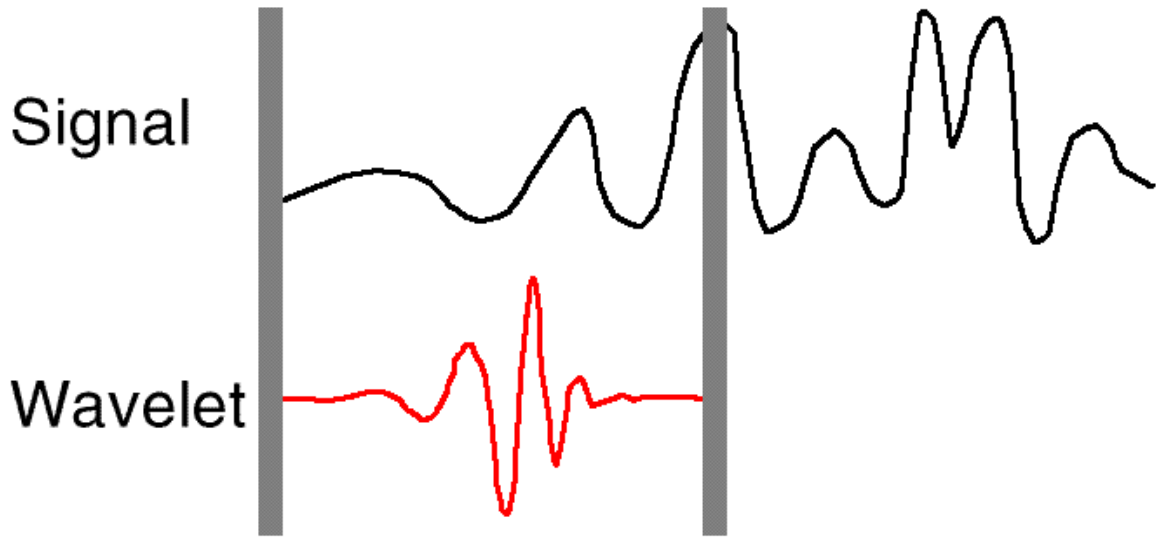
# CWT

- Step 3: Shift the wavelet to the right and repeat steps 1-2 until you've covered the whole signal

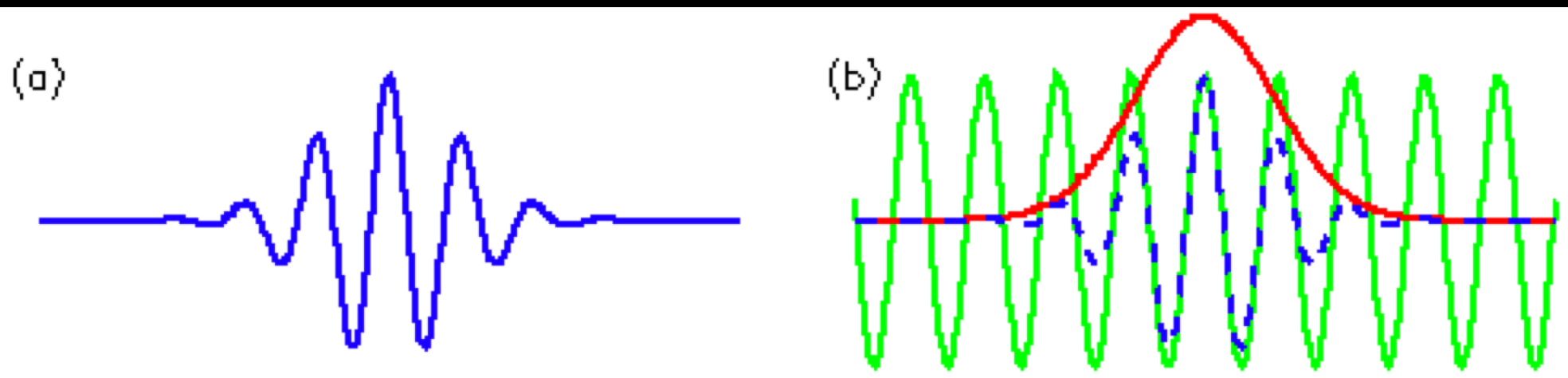


# CWT

- Step 4: Scale (stretch) the wavelet and repeat steps 1-3

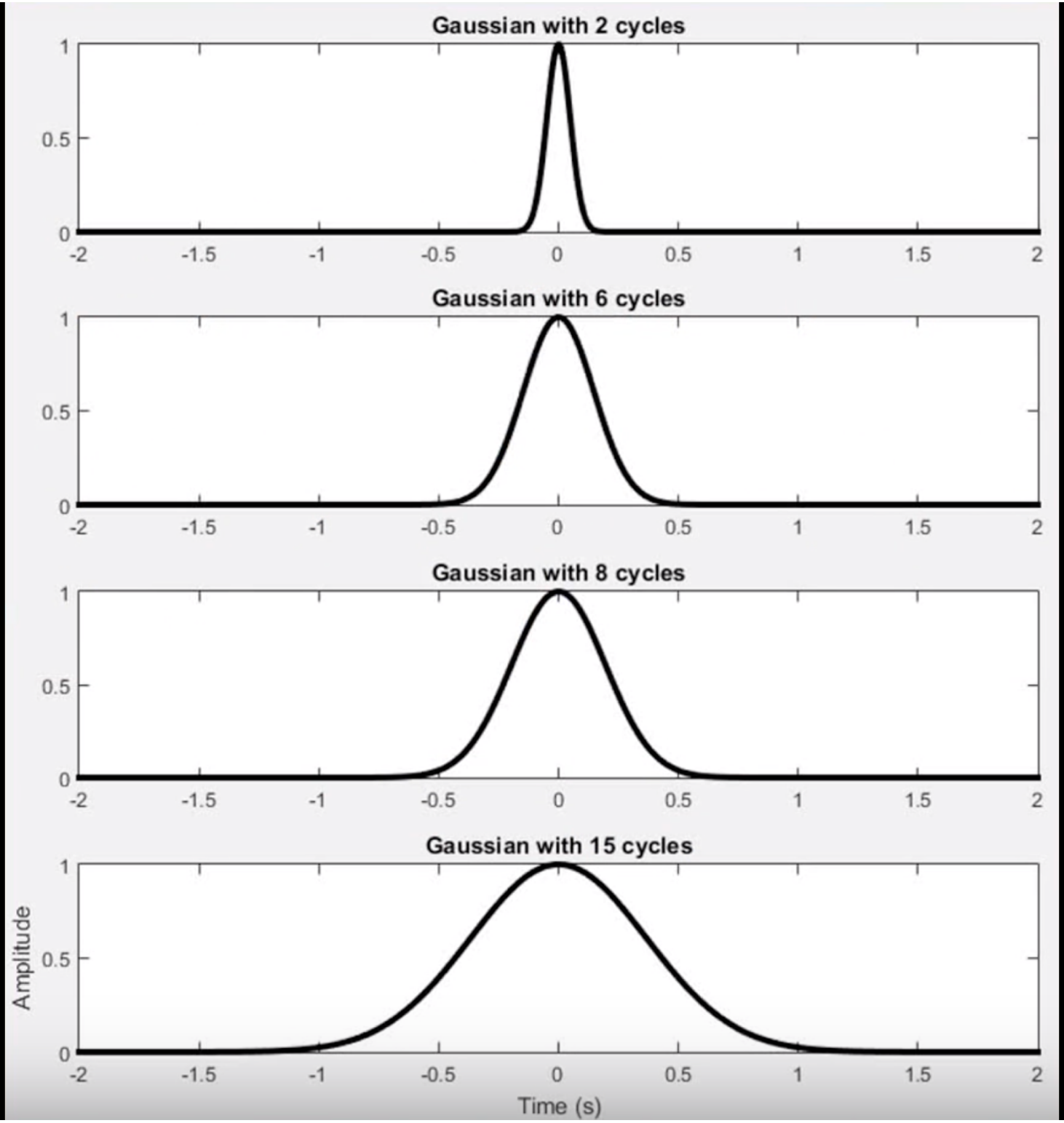


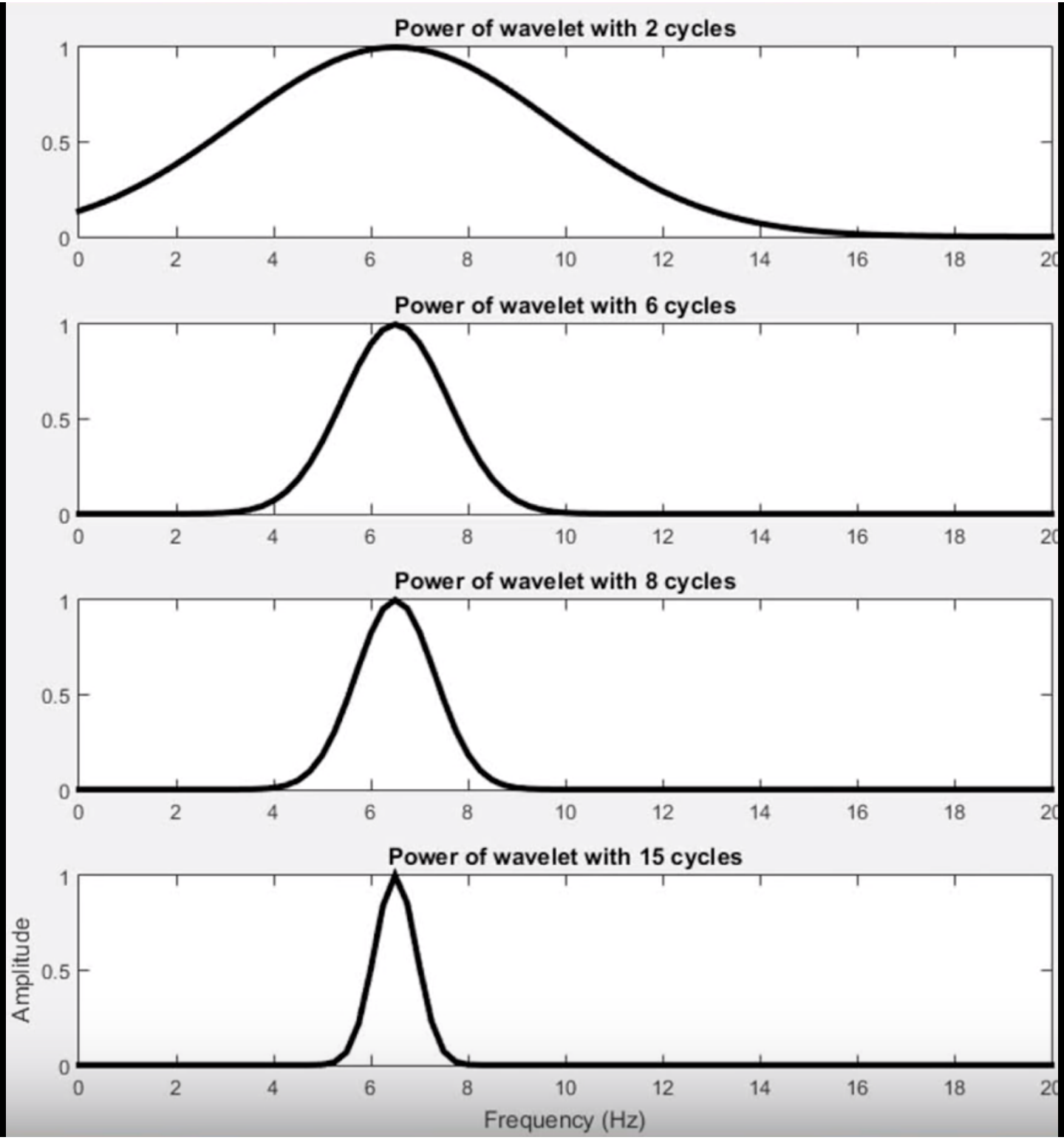
# The Morlet Parameter



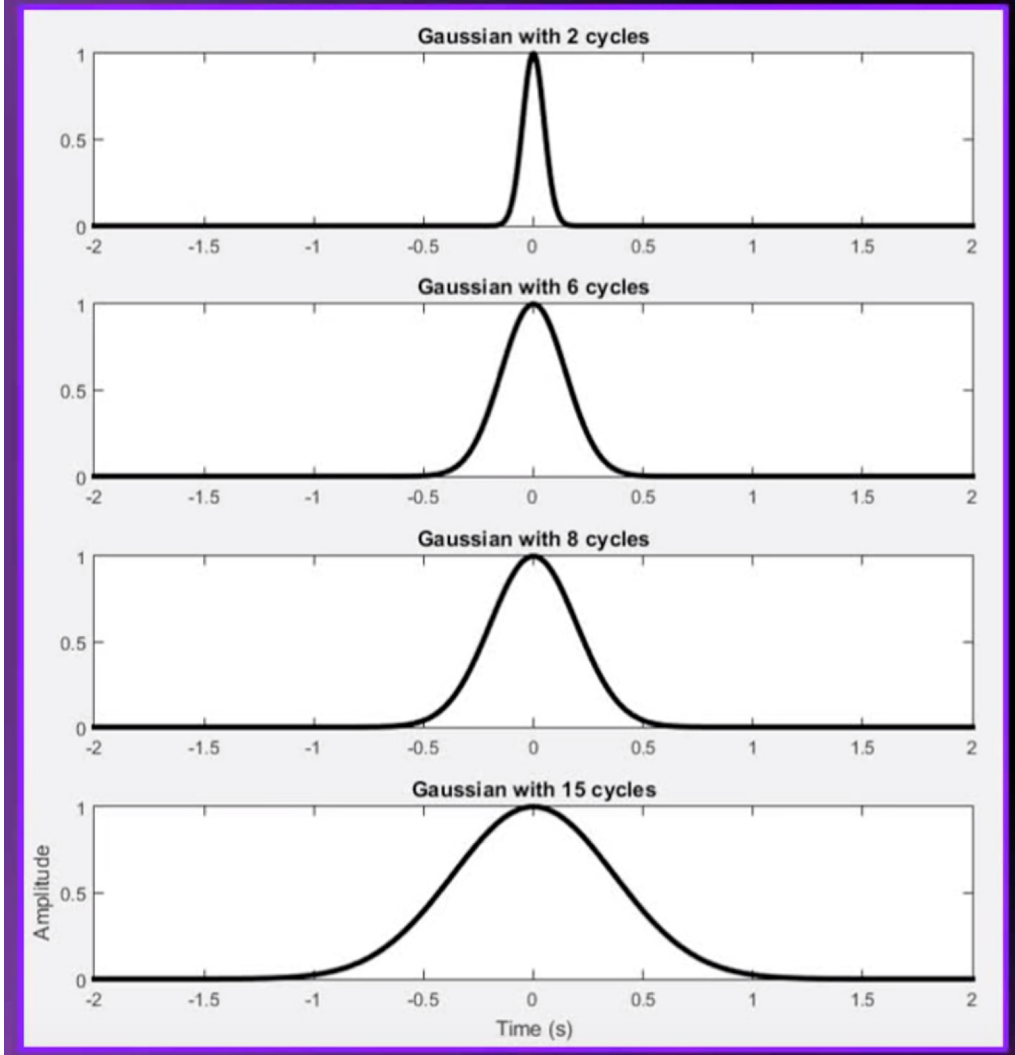

A wavelet is the convolution of a
Sine Wave and a Gaussian

```
% create wavelet and get its FFT
wavelet = exp(2*1i*pi*frequencies(fi).*time) .* ...
    exp(-time.^2./(2*(5/(2*pi*frequencies(fi)))^2));
```

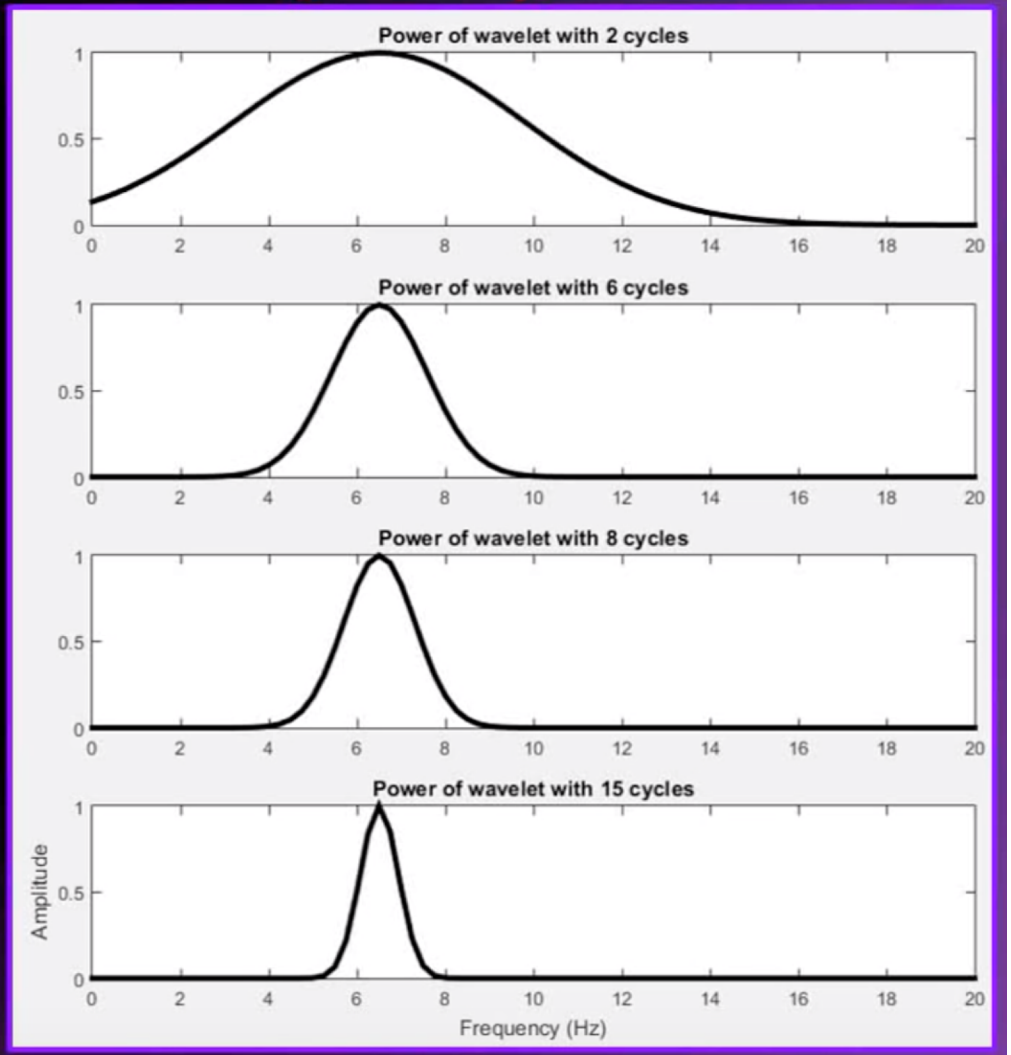

Gaussian with 2 cycles
Gaussian with 6 cycles
Gaussian with 8 cycles
Gaussian with 15 cycles
Amplitude
Time (s)
1
0.5
0
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2

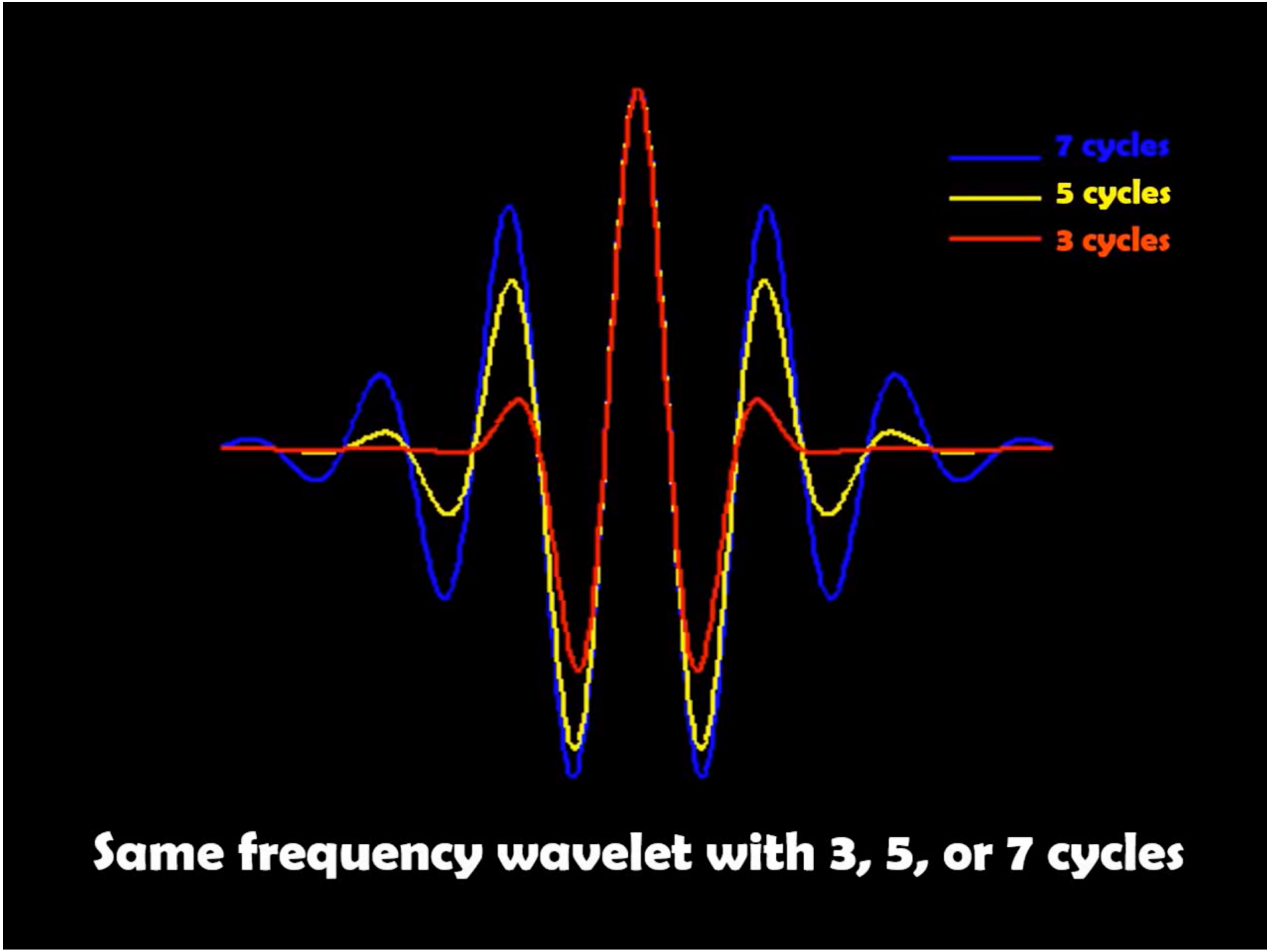

Power of wavelet with 2 cycles
Power of wavelet with 6 cycles
Power of wavelet with 8 cycles
Power of wavelet with 15 cycles
Amplitude
Frequency (Hz)
0
0.5
1
0
2
4
6
8
10
12
14
16
18
20

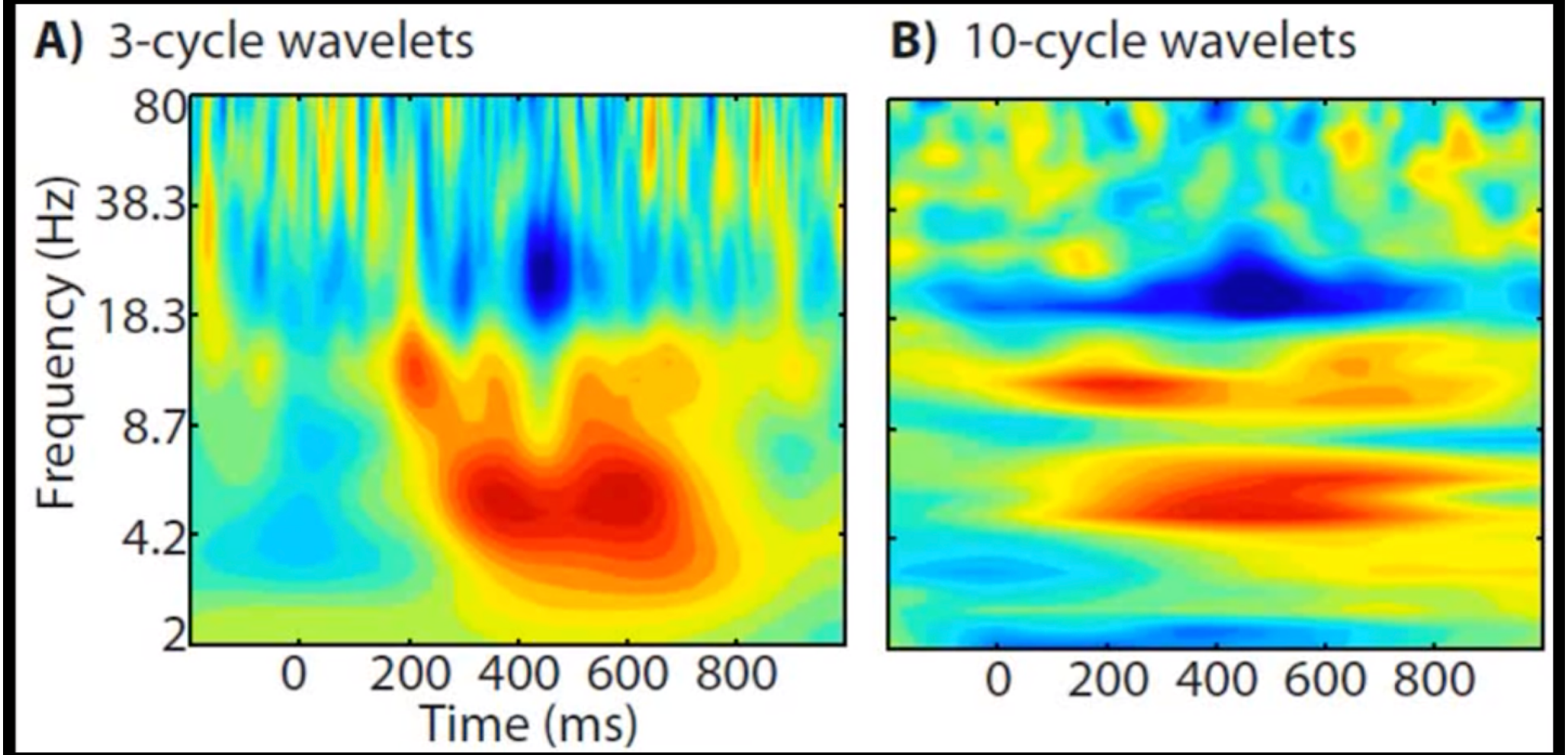
# Time domain

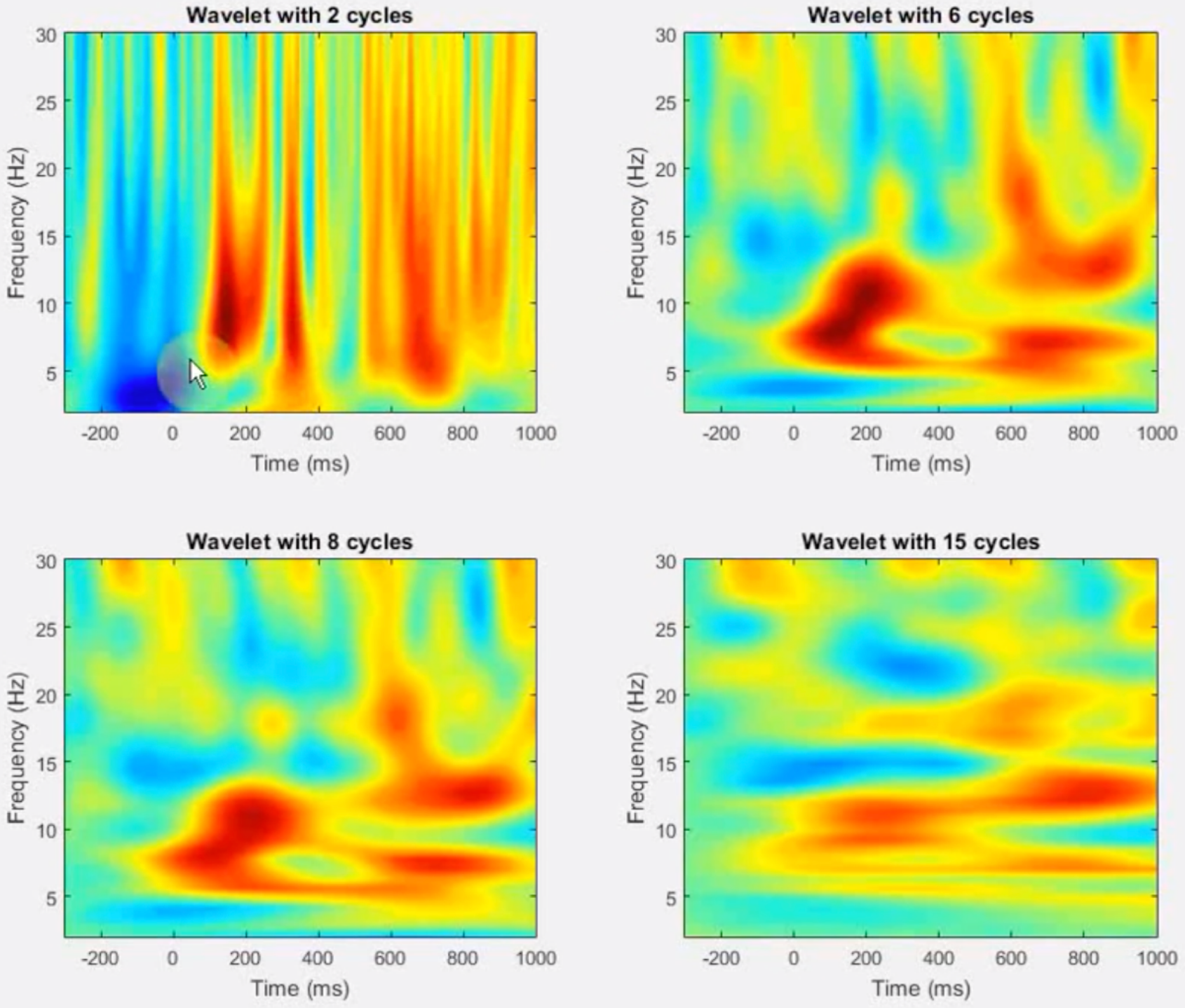



# Frequency domain




7 cycles
5 cycles
3 cycles
Same frequency wavelet with 3, 5, or 7 cycles


A) 3-cycle wavelets
B) 10-cycle wavelets
Frequency (Hz)
80
38.3
18.3
8.7
4.2
2
0
200
400
600
800
Time (ms)
0
200
400
600
800


Wavelet with 2 cycles
Wavelet with 6 cycles
Wavelet with 8 cycles
Wavelet with 15 cycles
Frequency (Hz)
Time (ms)
30
25
20
15
10
5
-200
0
200
400
600
800
1000

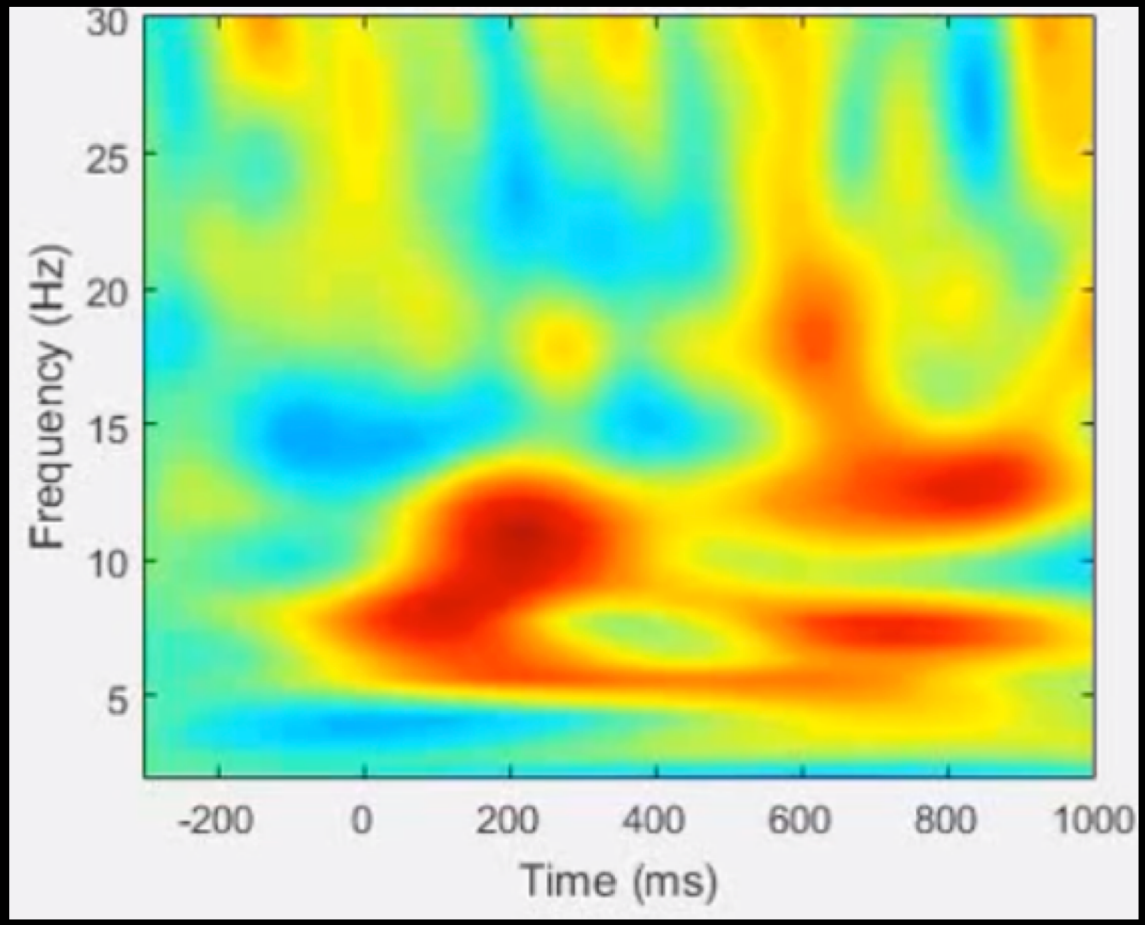
# Wavelets

Some argue this is the most accurate representation of the data because the activity reflected in the wavelet analysis is:

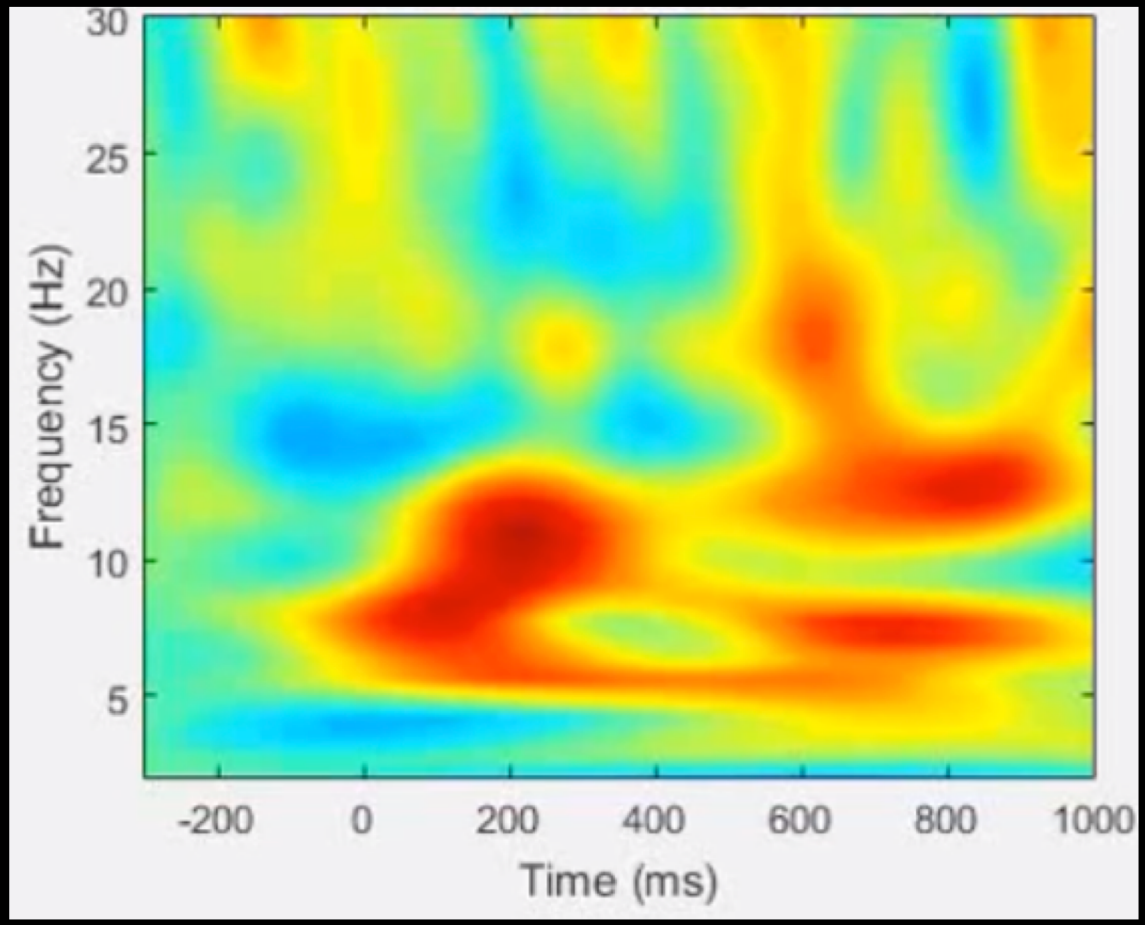
Evoked + Induced

The only real criticism is that for this to be a true statement there can be no edge artifacts – the wavelet window has to capture all of the data.

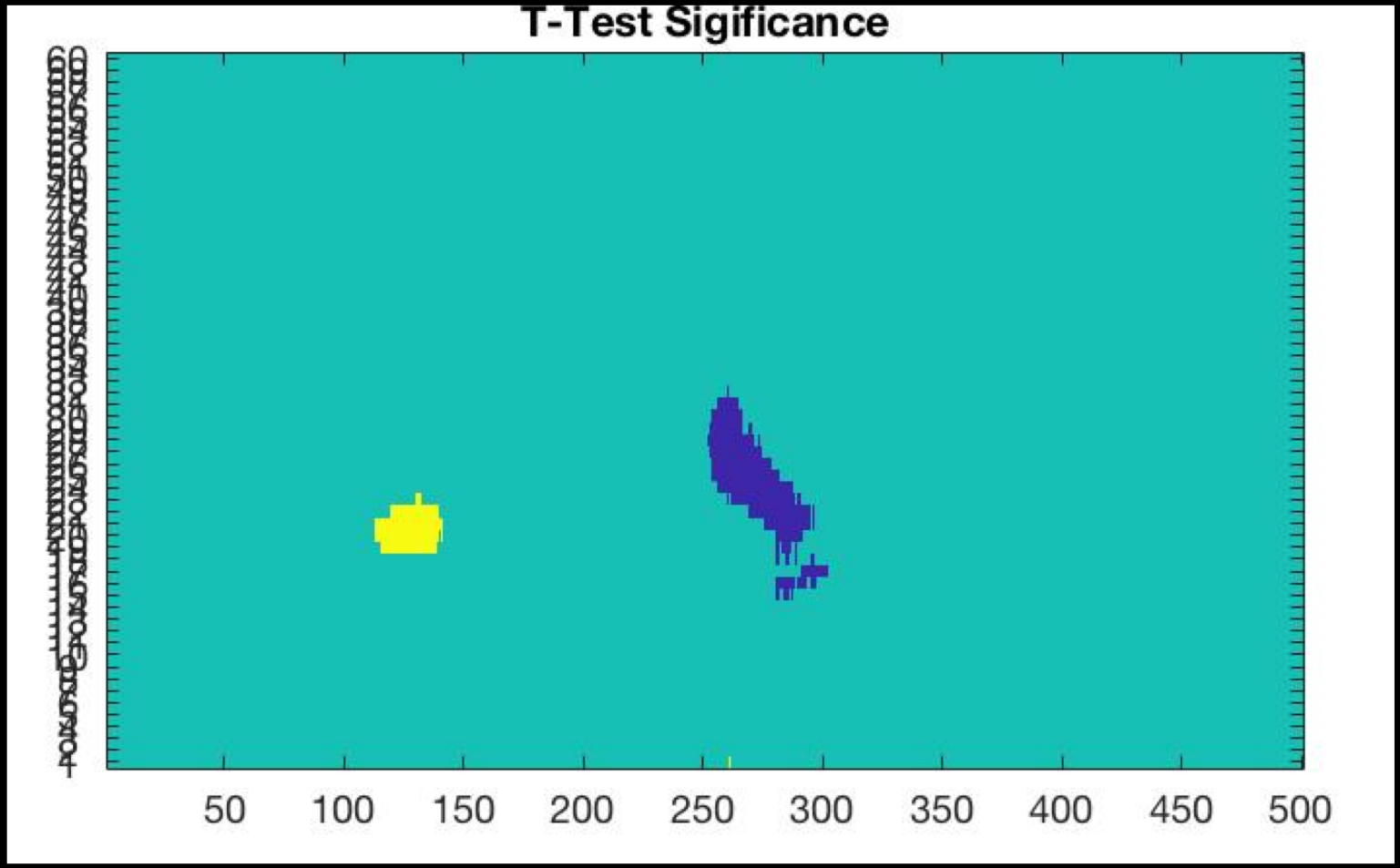
# Statistical Tests




How do you know what is significant?



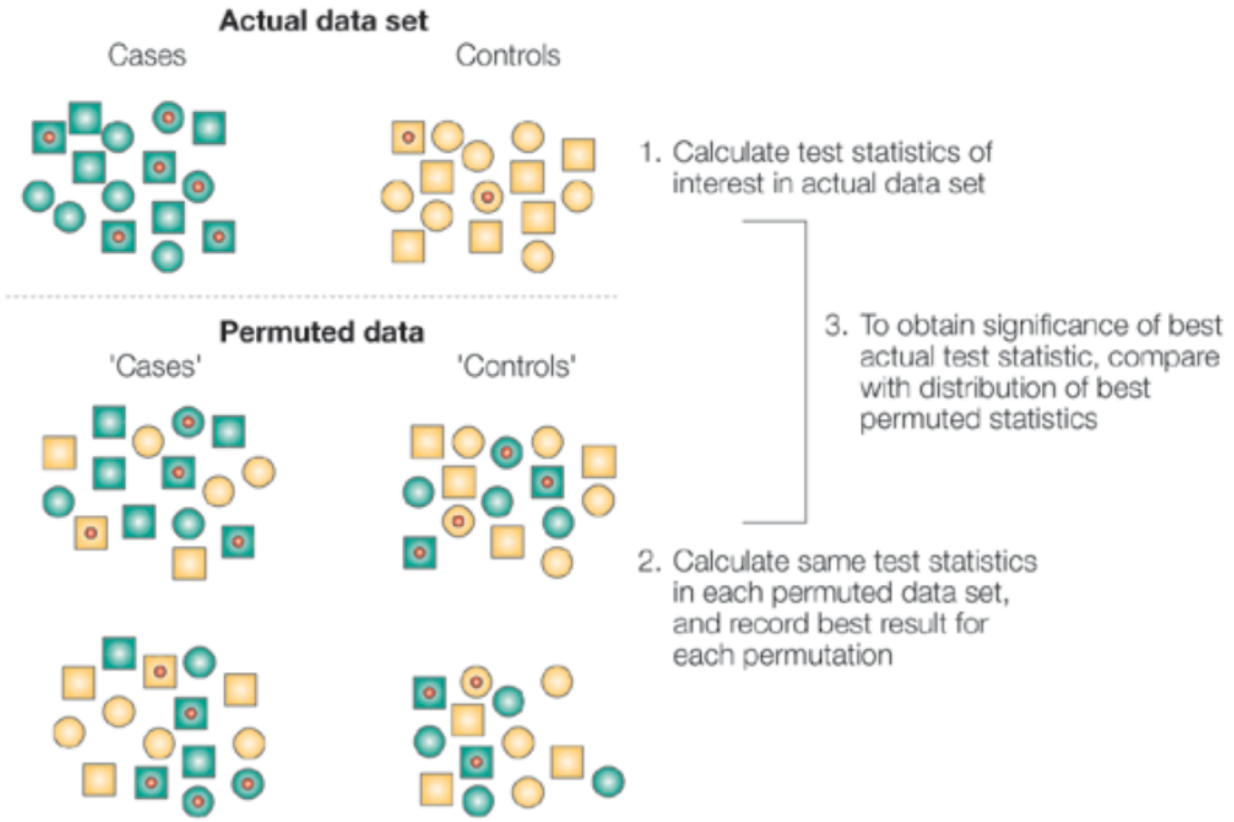

Solution 1: T-Test Every Point

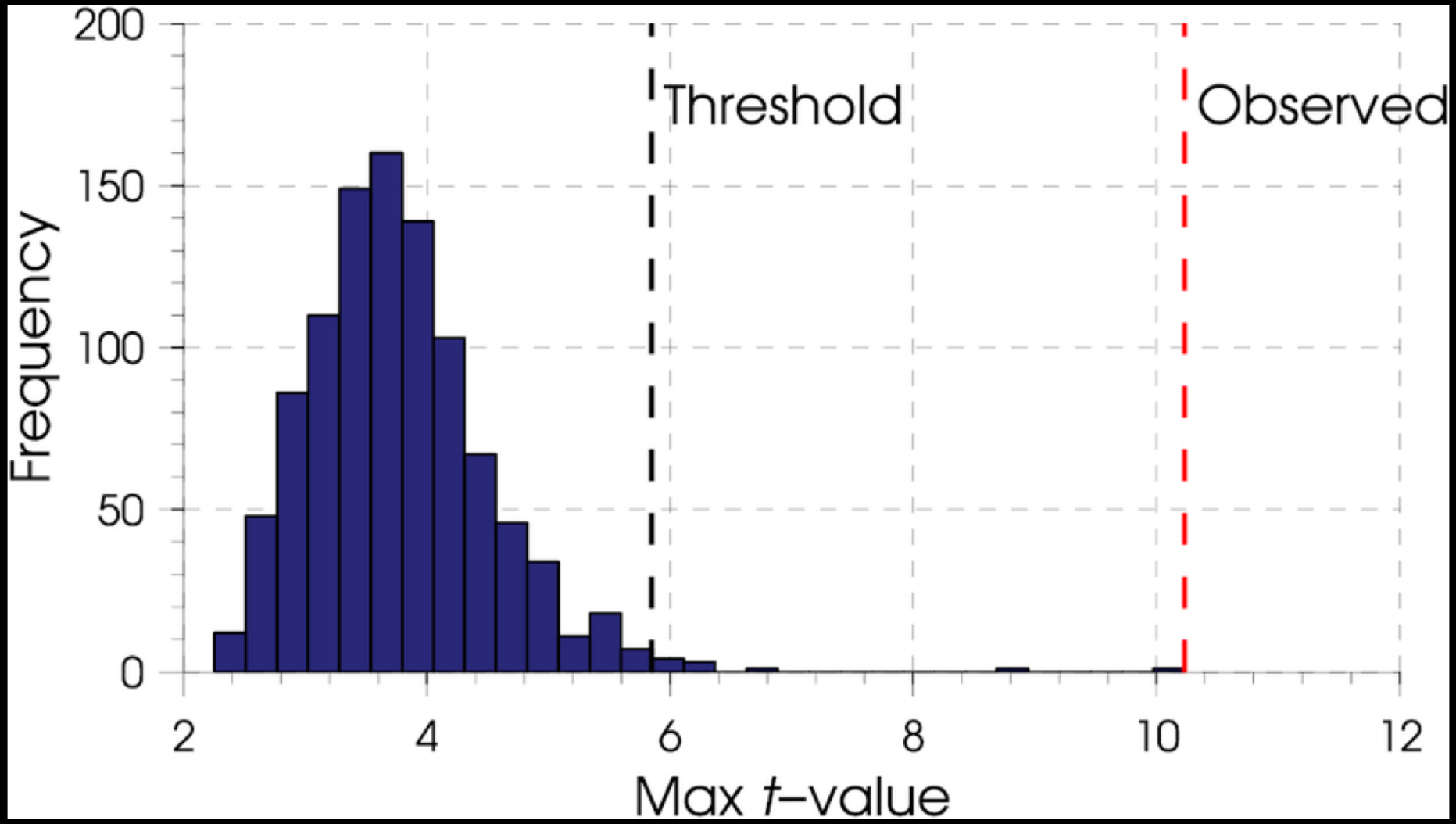

T-Test Sigificance
50
100
150
200
250
300
350
400
450
500

# Solution 2: Permutation Tests

# Permutation test

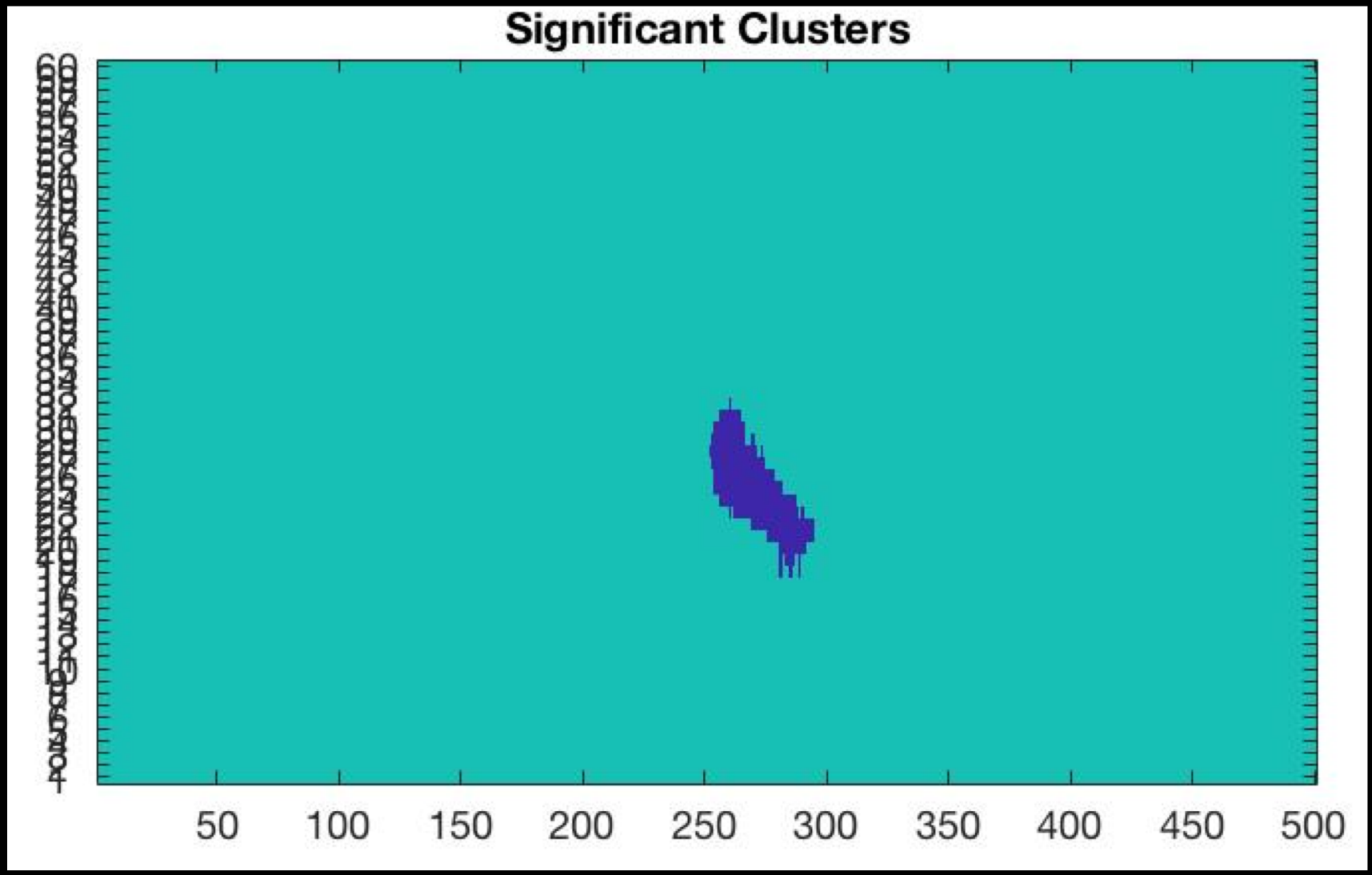
Response (y) variable is permuted to guarantee true $H_0$:




200
150
100
50
0
Frequency
Threshold
Observed
2
4
6
8
10
12
Max t–value

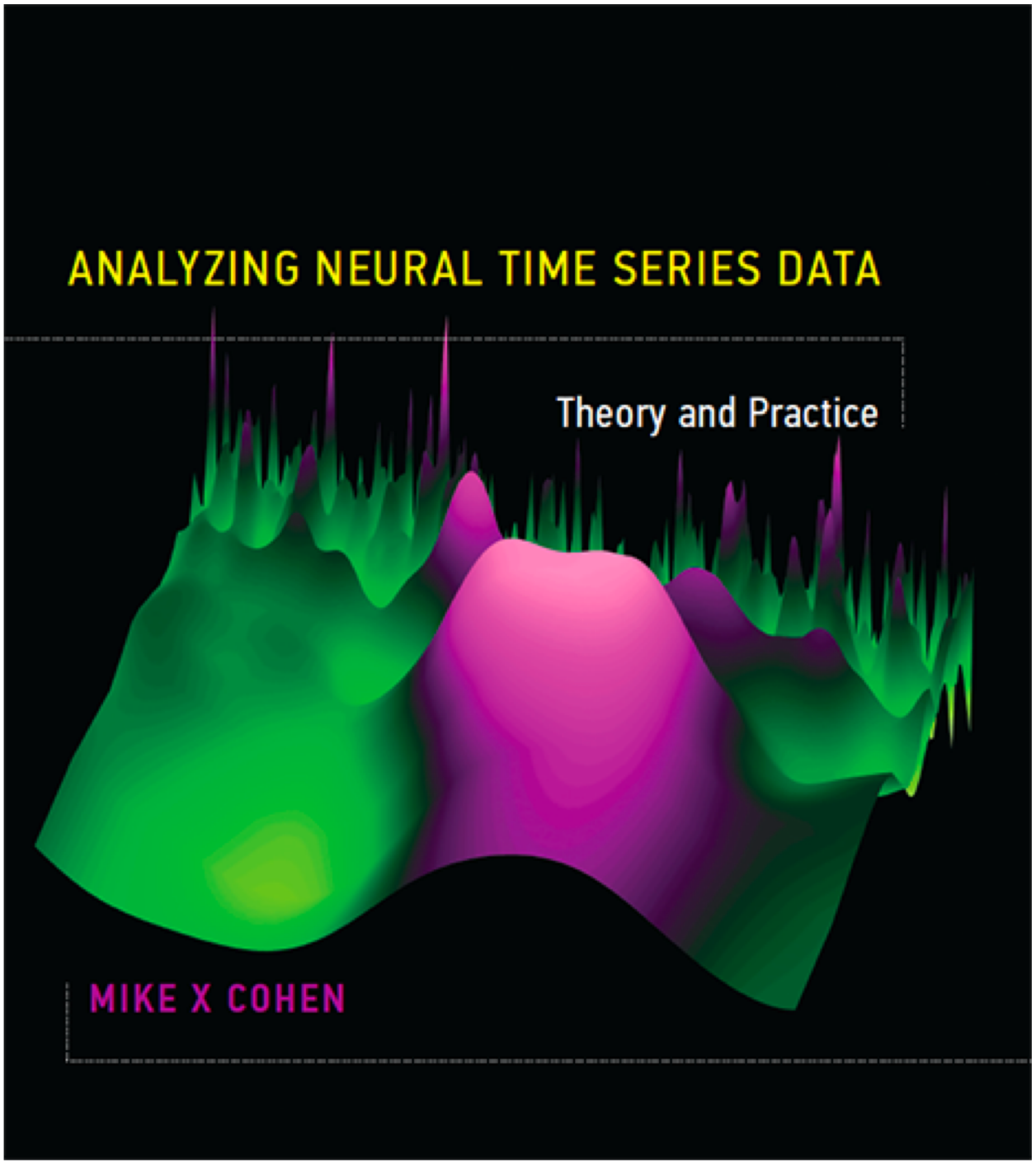
# Permutation test

- Calculate the test statistic for each permutation
  - 999 is a typical number
- P-value is the quantile of the real test statistic in the "empirical null distribution" of permutation test statistics
- Permutations tests *still have assumptions*:
  - samples are assumed to be independent and “exchangeable”
  - hidden structure such as families can cause anti-conservative p-values




You can also use a cluster based correction with a permutation test.

# ANALYZING NEURAL TIME SERIES DATA



# Wavelet Demo