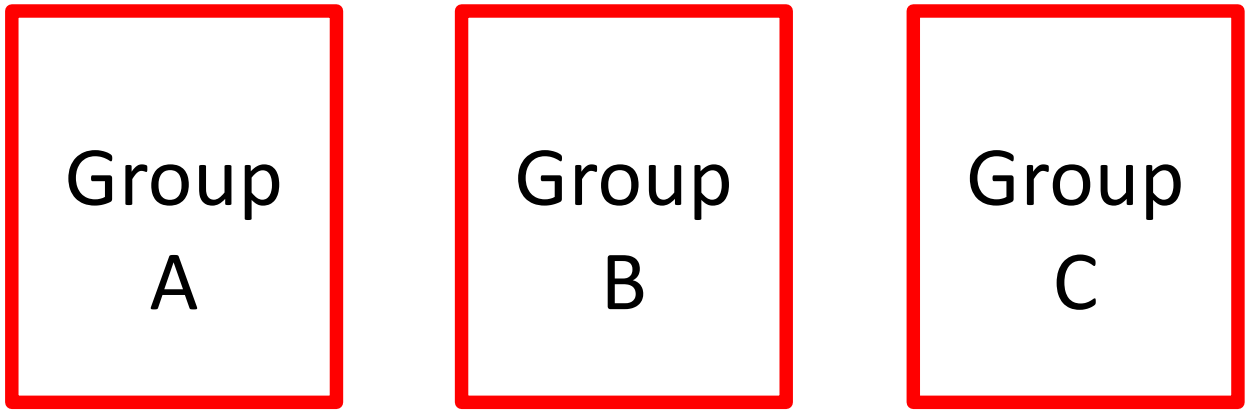
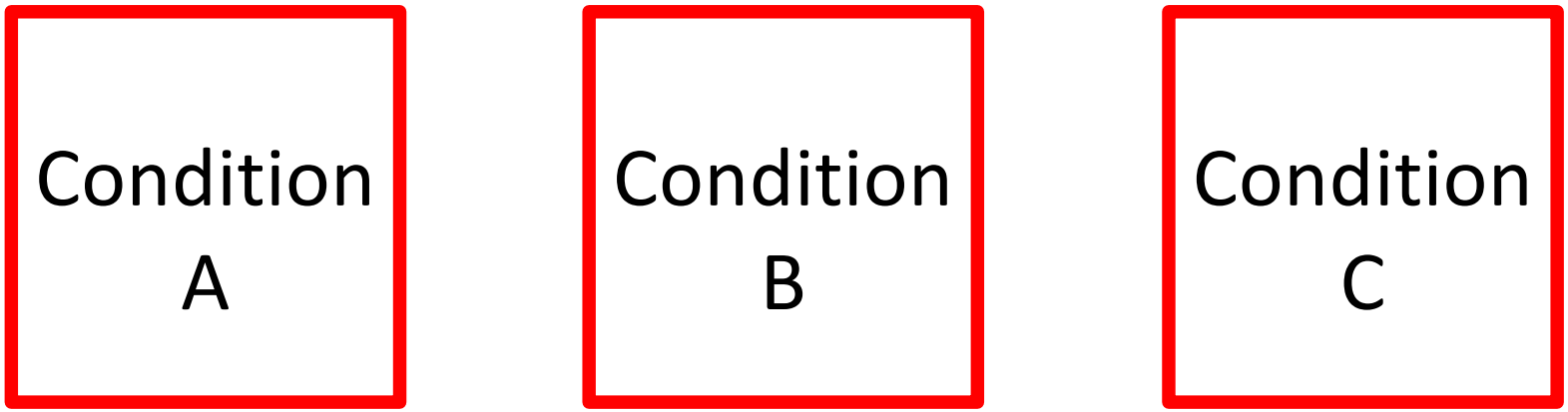
Repeated Measures Analysis of Variance

Review

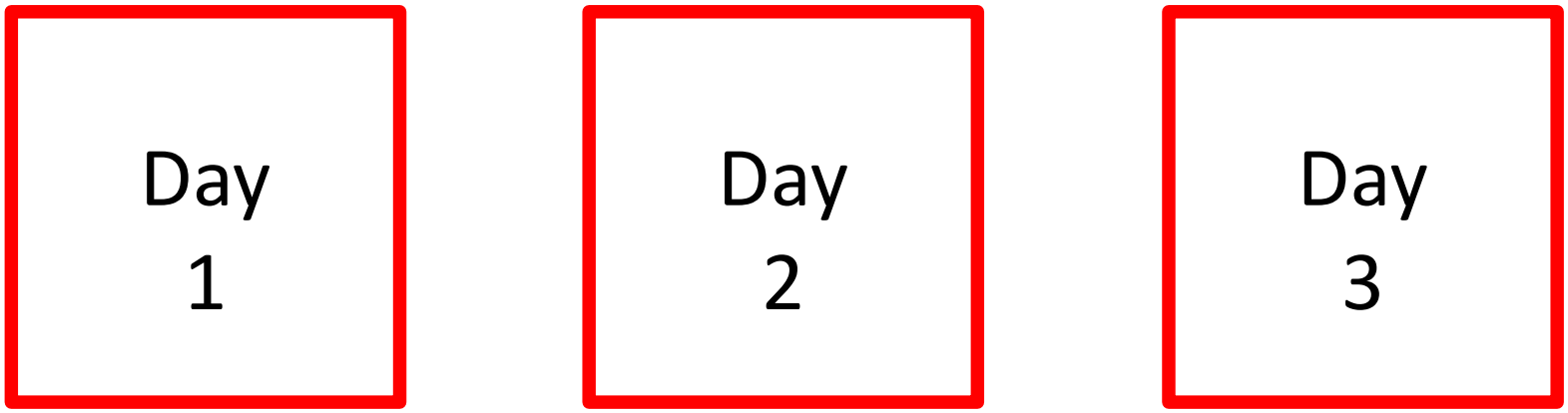
Univariate Analysis of Variance



Repeated Measures Analysis of Variance



Repeated Measures Analysis of Variance



Basic Logic of RM ANOVA

Hypothesis Testing

$H_o : \upsilon_1 = \upsilon_2 = \upsilon_3$

$H_1 : \upsilon_1 \neq \upsilon_2 \neq \upsilon_3$ (at least one difference)

Basic Logic of RM ANOVA

$$F = \frac{MS_{conditions}}{MS_{error}}$$

Variance explained by treatment

Variance explained by error

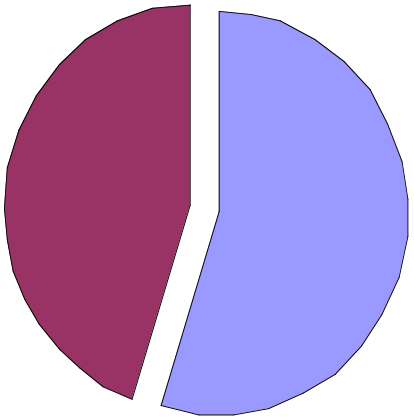
Basic Logic of RM ANOVA

$$F = \frac{MS_{conditions}}{MS_{error}}$$

Note, $Ms_{conditions}$ is the same as $Ms_{between}$, whats different is the error term.

Between Subjects ANOVA

Unexplained Variance (Error)

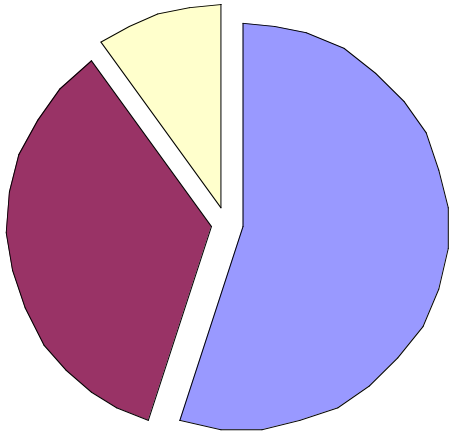


Explained Variance

Within Subjects ANOVA (Repeated Measures)

Subjects Variance

Unexplained Variance (Error)



Explained Variance

Recall, between subjects ANOVA

$$SS_{total} = SS_{between} + SS_{within}$$

Deviations of group means from the grand mean

Deviations of subject scores from the cell mean

$$SS_{total} = SS_{conditions} + SS_{subjects} + SS_{error}$$

Deviations of condition MEANS from the grand mean

Deviations of subject MEANS from the grand mean

An Example

Repeated Measures ANOVA

	Condition					
P	**One**	**Two**	**Three**			
1	**4**	**4**	**6**			
2	**3**	**4**	**6**			
3	**4**	**3**	**5**			
4	**3**	**5**	**5**			
5	**3**	**4**	**7**			

Repeated Measures ANOVA

	Condition					
P	One	Two	Three	$\overline{x}$		
1	4	4	6	4.666		
2	3	4	6	3.666		
3	4	3	5	4		
4	3	5	5	4.333		
5	3	4	7	4.666		
$\overline{x}$	3.4	4	5.4			
					$\overline{x}_{..}$	4.266

Recall…

$$SS = (x - \overline{x}_{GM})^2$$

$SS_{conditions}$

$$SS_{conditions} = n\sum(\overline{x}_{conditions} - \overline{\overline{x}}_{..})^2$$

$$SS_{conditions} = 5[(3.4 - 4.266)^2 + ...]$$

$$SS_{conditions} = 10.533$$

$SS_{subjects}$

$$SS_{subjects} = k\sum(\overline{x}_{subjects} - \overline{\overline{x}}_{..})^2$$

$$SS_{subjects} = 3[(4.666 - 4.266)^2 + ...]$$

$$SS_{subjects} = 2.266$$

$$SS_{error}$$

$$SS_{error} = SS_{total} - SS_{conditions} - SS_{subjects}$$

$$SS_{error} = 18.933 – 10.533 – 2.266$$

$$SS_{error} = 6.133$$

Recall…

$$MS = \frac{SS}{df}$$

Degrees of Freedom

	Condition		
P	**One**	**Two**	**Three**
1	**4**	**4**	**6**
2	**3**	**4**	**6**
3	**4**	**3**	**5**
4	**3**	**5**	**5**
5	**3**	**4**	**4**

$$df_{total} = N - 1$$

Degrees of Freedom

	Condition		
P	**One**	**Two**	**Three**
1	**4**	**4**	**6**
2	**3**	**4**	**6**
3	**4**	**3**	**5**
4	**3**	**5**	**5**
5	**3**	**4**	**4**

$$df_{conditions} = k - 1$$

Degrees of Freedom

	Condition		
P	**One**	**Two**	**Three**
1	**4**	**4**	**6**
2	**3**	**4**	**6**
3	**4**	**3**	**5**
4	**3**	**5**	**5**
5	**3**	**4**	**4**

$$df_{subjects} = n - 1$$

Degrees of Freedom

	Condition		
P	**One**	**Two**	**Three**
1	**4**	**4**	**6**
2	**3**	**4**	**6**
3	**4**	**3**	**5**
4	**3**	**5**	**5**
5	**3**	**4**	**4**

$$df_{error} = df_{conditions} * df_{subjects}$$

Repeated Measures ANOVA Summary Table

Source	**df**	**SS**	**MS**	**F**
Subjects	**n-1**	$SS_{subjects}$		
Conditions	**k-1**	$SS_{conditions}$	$\frac{SS_{conditions}}{df_{conditions}}$	$\frac{MS_{conditions}}{MS_{error}}$
Error	**(n-1)*(k-1)**	SS_{error}	$\frac{SS_{error}}{df_{error}}$	
Total	**N-1**	SS_{total}		

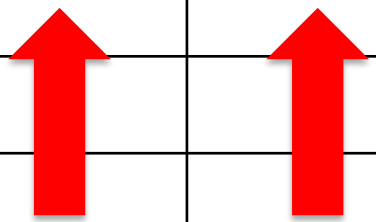
Repeated Measures ANOVA Summary Table

Source	df	SS	MS	F
Subjects	**4**	2.266		
Conditions	**2**	10.533	5.267	6.870
Error	**8**	6.133	0.767	
Total	**14**	18.933		

Post-Hoc Comparisons: Simple Effects Analysis

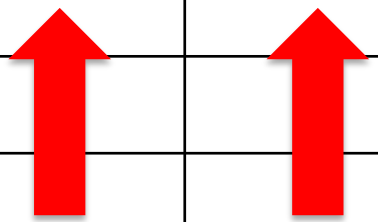
Repeated Measures ANOVA

	Condition					
P	One	Two	Three			
1	4	4	6			
2	3	4	6			
3	4	3	5			
4	3	5	5			
5	3	4	7			



Repeated Measures ANOVA

	Condition					
P	One	Two	Three			
1	4	4	6			
2	3	4	6			
3	4	3	5			
4	3	5	5			
5	3	4	7			



Repeated Measures ANOVA

	Condition					
P	One	Two	Three			
1	4	4	6			
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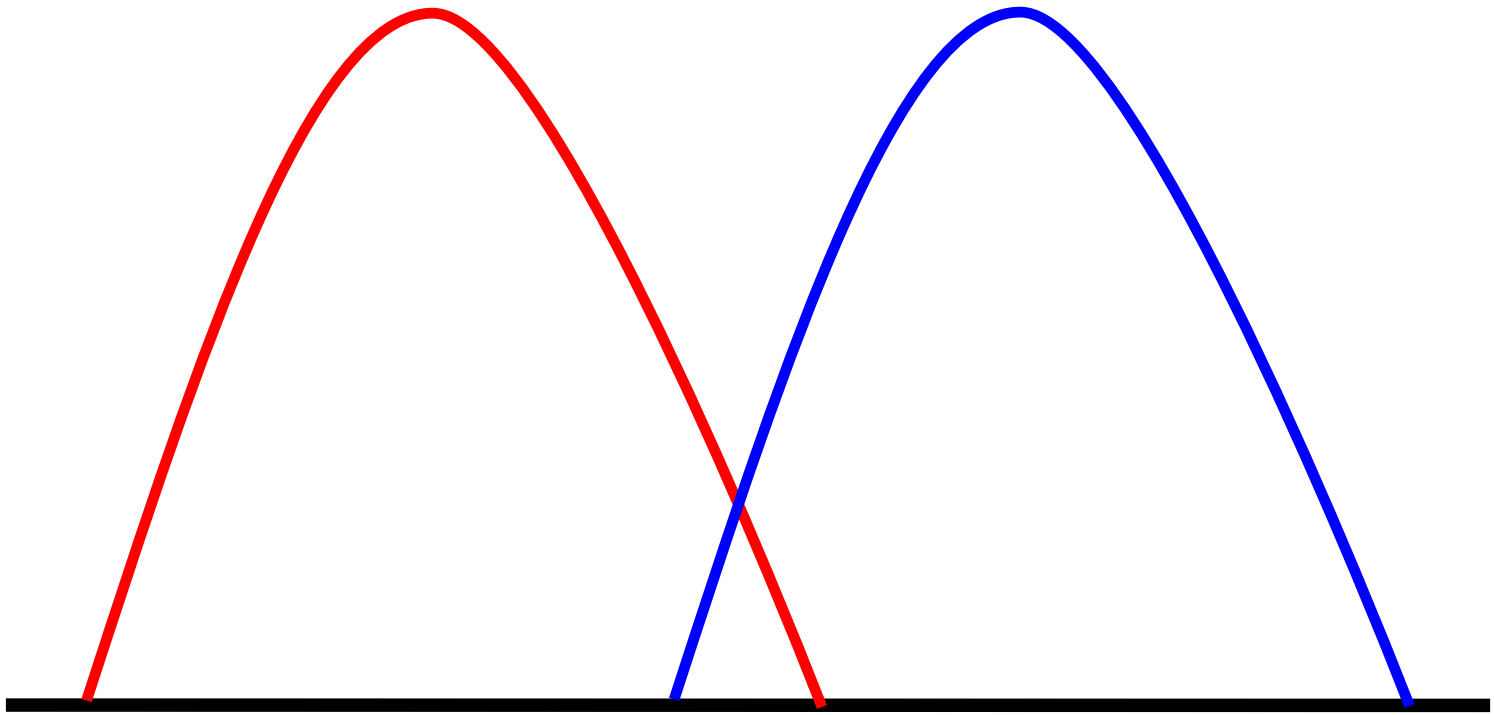
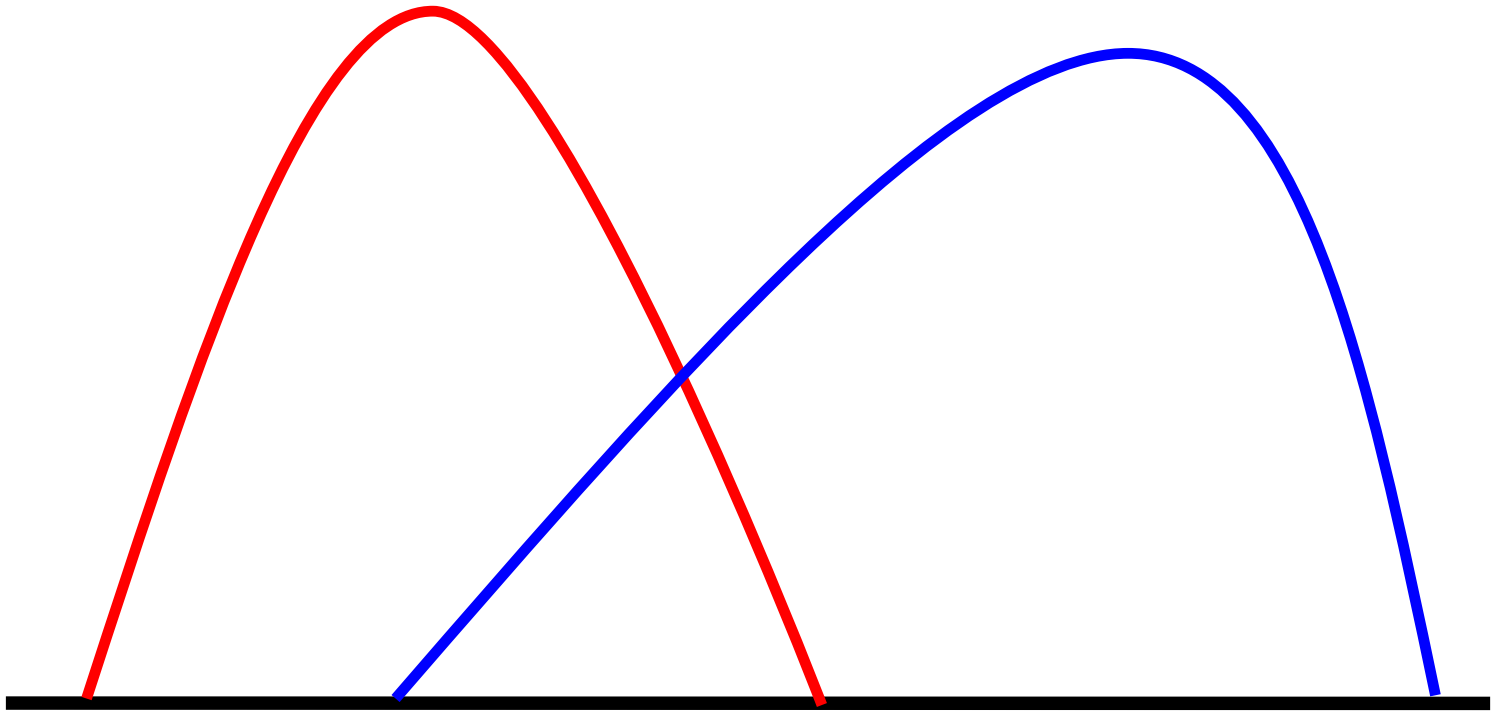
Assumptions of repeated measures ANOVA

1. Normality



Assumptions of repeated measures ANOVA

1. Normality



Assumptions of repeated measures ANOVA

2. Homogeneity of Variance

$$\sigma_1^2 = \sigma_2^2 = \sigma_3^2$$

3. The Assumption of Sphericity

Correlations among pairs of variables are equal…

NO!

Sphericity

- Sphericity is the property that the covariance of the difference scores of the IV levels are same
- Violations generally lead to inflated F statistics (and hence inflated Type I error).

Sphericity
Maulchy's Test

What does it mean?

Effect	DFn	DFd	F	p	ges
condition	2	30	4.473097	0.01994544	* 0.07827009

Mauchly's Test for Sphericity

Effect	W	p
condition	0.9392082	0.6446637

Sphericity Corrections

Effect	GGe	p[GG]	HFe	p
condition	0.942692	0.02222867	1.073846	0.01994544

Okay, if the sphericity test is not significant…

Keep on going…

Okay, if the sphericity test is significant…

1) Check epsilon. The epsilon means the departure from the sphericity, in other words, how far the data is from the ideal sphericity.

   The epsilon is a number between 0 and 1, if the epsilon is equal to 1, the data have sphericity.

Look at…

$ANOVA

	Effect	DFn	DFd	F	p	p<.05	ges
2	condition	2	30	4.473097	0.01994544	*	0.07827009

$`Mauchly's Test for Sphericity`

	Effect	W	p	p<.05
2	condition	0.9392082	0.6446637	

$`Sphericity Corrections`

	Effect	GGe	p[GG]	p[GG]<.05	HFe	p[HF]	p[HF]<.05
2	condition	**0.942692**	0.02222867	*	**1.073846**	0.01994544	*

Which one should I look at?

Generally, Greenhouse-Geisser.

BUT… if GG epsilon > 0.75

USE Huynh-Feldt.

WHY? GG tends to be too strict when epsilon is large.

ALSO, use Huynh-Feldt when n is small (less than 15)

What do I do with it?

The test provides you with the corrected p value.

Look at…

$ANOVA

	Effect	DFn	DFd	F	p	p<.05	ges
2	condition	2	30	4.473097	0.01994544	*	0.07827009

$`Mauchly's Test for Sphericity`

	Effect	W	p	p<.05
2	condition	0.9392082	0.6446637	

$`Sphericity Corrections`

	Effect	GGe	p[GG]	p[GG]<.05	HFe	p[HF]	p[HF]<.05
2	condition	**0.942692**	0.02222867	*	**1.073846**	0.01994544	*

But you also have to…

Correct df's… (effect and error term)

Multiply then by Epsilon:

2 * **0.942692** **= 1.885**

Assumptions of repeated measures ANOVA

Mauchly's Test of Sphericity[b]

Measure:MEASURE_1

Within Subjects Effect	Mauchly's W	Approx. Chi-Square	df	Sig.	Epsilon[a]		
					Greenhouse-Geisser	Huynh-Feldt	Lower-bound
Condition	.720	.657	2	.720	.781	1.000	.500

Tests of Within-Subjects Effects

Measure:MEASURE_1

Source		Type III Sum of Squares	df	Mean Square	F	Sig.
Condition	Sphericity Assumed	10.533	2	5.267	6.870	.018
	Greenhouse-Geisser	10.533	1.931	5.456	6.870	.020
	Huynh-Feldt	10.533	2.000	5.267	6.870	.018
	Lower-bound	10.533	1.000	10.533	6.870	.059
Error(Condition)	Sphericity Assumed	6.133	8	.767		
	Greenhouse-Geisser	6.133	7.723	.794		
	Huynh-Feldt	6.133	8.000	.767		
	Lower-bound	6.133	4.000	1.533		

But what is SPERICITY?

Variance

$$s^2 = \frac{\sum (X - \overline{X})^2}{N - 1}$$

Covariance

The degree to which two variables vary together.

$$COV_{xy} = \frac{\sum (x - \bar{x})(y - \bar{y})}{N - 1}$$

Covariance

The degree to which two variables vary together.

1	1
2	2
3	3
4	4

COV = 1.25

1	1
2	1
3	1
4	1

COV = 0

1	4
2	3
3	2
4	1

COV = -1.25

1	2
2	4
3	9
4	16

COV = 5.875

Assumptions of repeated measures ANOVA

3. Sphericity

	Condition			
	One	Two	Three	Four
One	S_1^2			
Two		S_2^2		
Three			S_3^2	
Four				S_4^2

Assumptions of repeated measures ANOVA

3. Sphericity

	Condition			
	One	Two	Three	Four
One	S_1^2	S_{12}	S_{13}	S_{14}
Two	S_{21}	S_2^2	S_{23}	S_{24}
Three	S_{31}	S_{32}	S_3^2	S_{34}
Four	S_{41}	S_{42}	S_{43}	S_4^2

Assumptions of repeated measures ANOVA

3. Sphericity: <u>Compound Symmetry</u>

	Condition			
	One	Two	Three	Four
One	S_1^2	S_{12}	S_{13}	S_{14}
Two	S_{21}	S_2^2	S_{23}	S_{24}
Three	S_{31}	S_{32}	S_3^2	S_{34}
Four	S_{41}	S_{42}	S_{43}	S_4^2

The variances AND covariances are equal

Assumptions of repeated measures ANOVA

3. Sphericity: Difference Scores

	Condition		
P	C1-C2	C1-C3	C1-C4
1	x_{11}-x_{12}	x_{11}-x_{13}	x_{11}-x_{14}
2	x_{21}-x_{22}	x_{21}-x_{23}	x_{21}-x_{24}
3	x_{31}-x_{32}	x_{31}-x_{33}	x_{31}-x_{34}
4	x_{41}-x_{42}	x_{41}-x_{43}	x_{41}-x_{44}

The variances of the difference scores are equal

Assumptions of repeated measures ANOVA

3. Sphericity: Covariance Matrix

$$S^2_{x-y} = S^2_x + S^2_y - 2S_{xy}$$

The variances of the difference scores are equal

Factorial Repeated Measures ANOVA

An Example

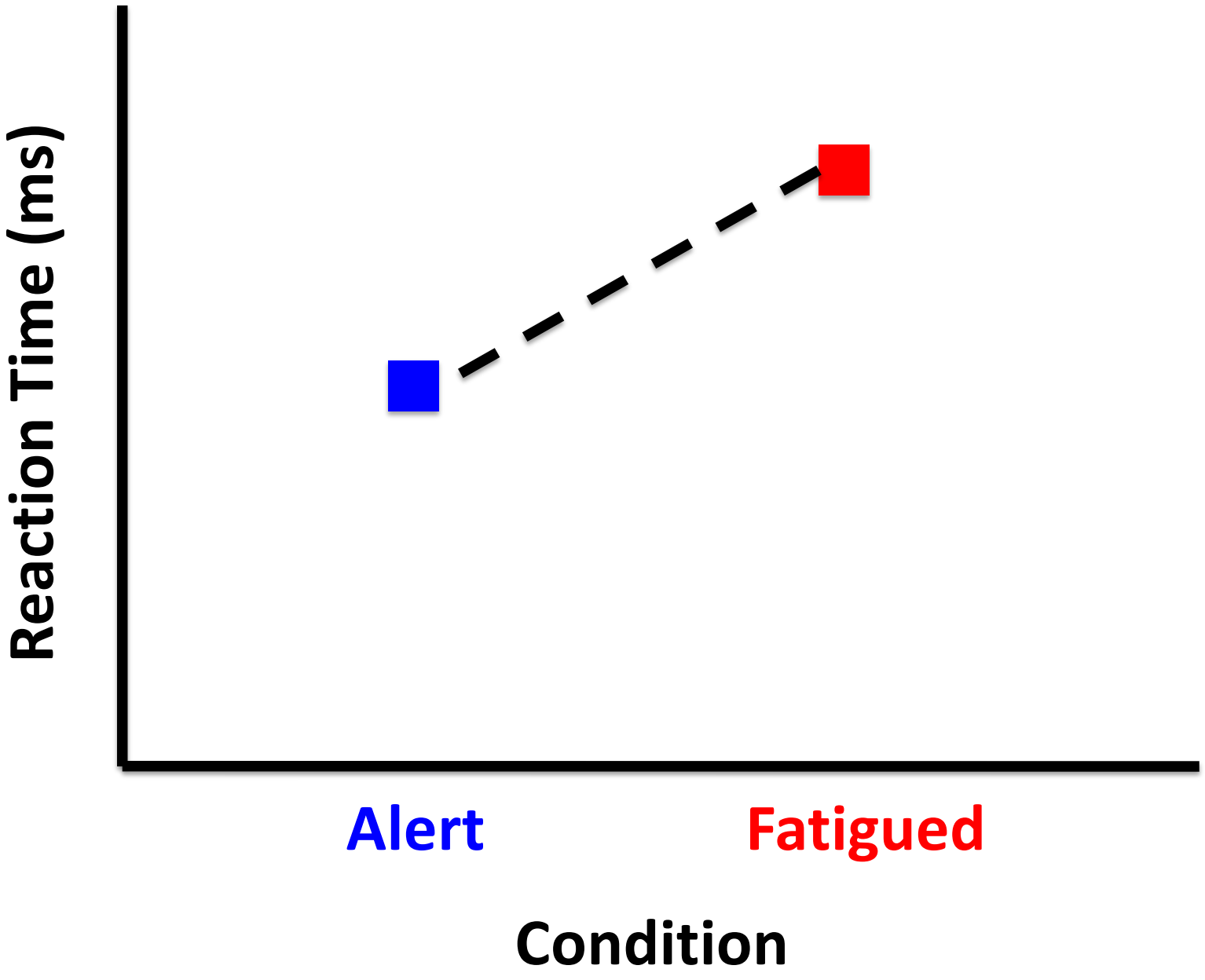
Participants in an experiment are asked to perform a cued reaction time task when they are alert and when they are fatigued. As such, you have participants performing a reaction time task with three conditions (valid cue, no cue, invalid cue) when they are either alert or fatigued.

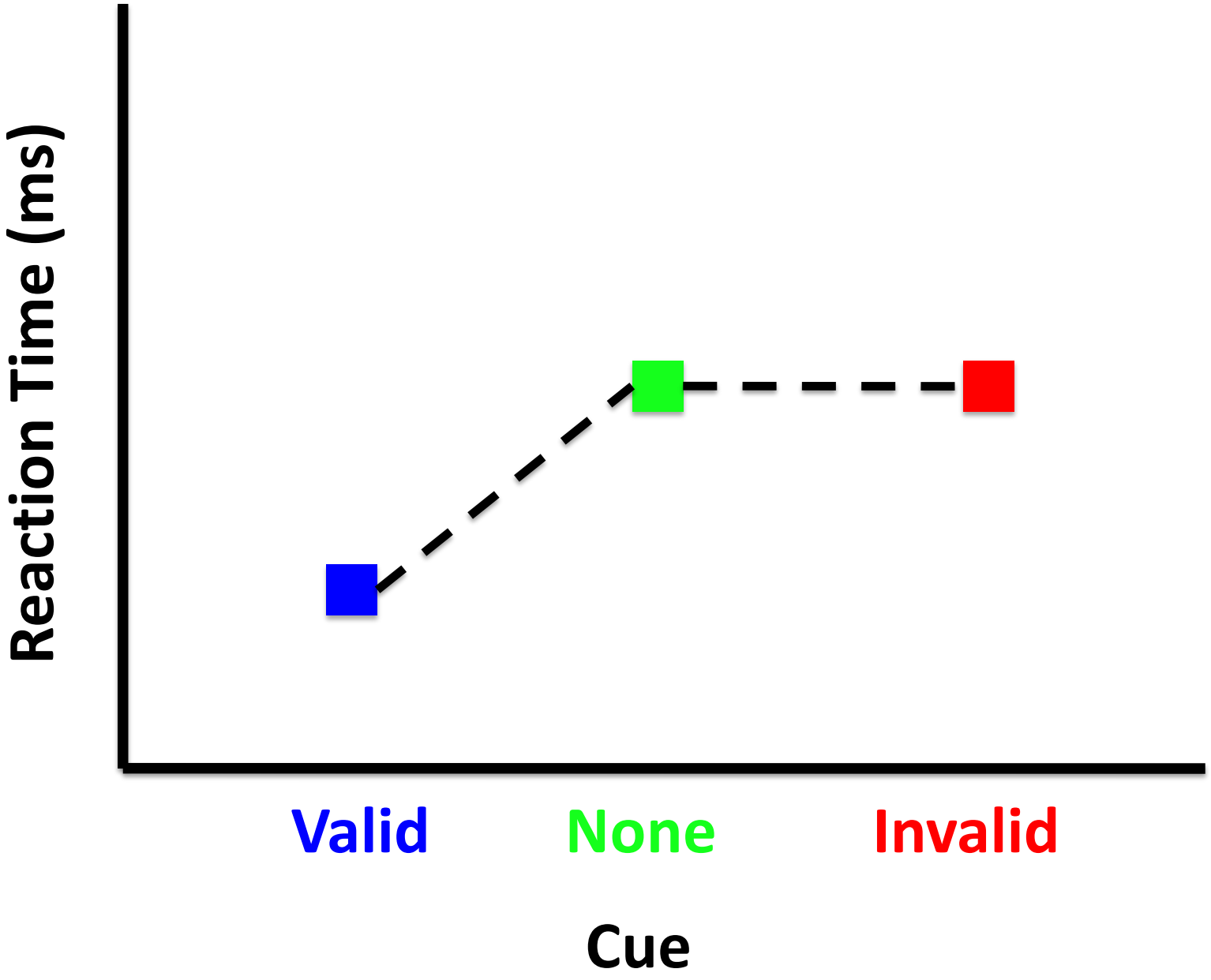
An Example

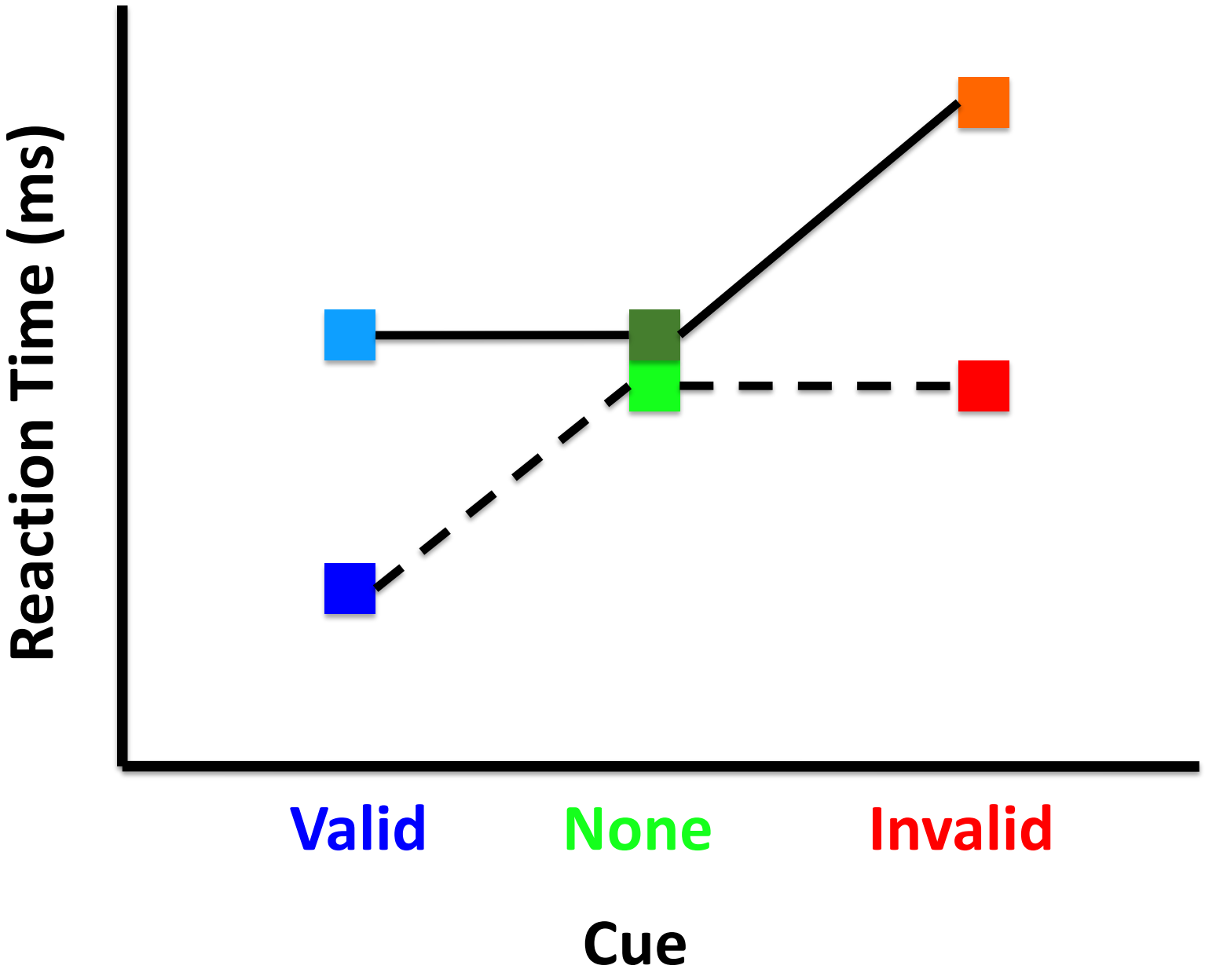
Main Effect: Fatigue (Alert, Fatigued)

Main Effect: Condition (Valid Cue, No Cue, Invalid Cue)

Interaction: Fatigue x Condition


Reaction Time (ms)
Alert
Fatigued
Condition


Reaction Time (ms)
Valid
None
Invalid
Cue


Reaction Time (ms)
Valid
None
Invalid
Cue