

# EPHE 591

Post Hoc Comparisons and Contrasts

Field, Chapter 10

# Basic Logic

Recall that ANOVA tests the following null hypothesis:

$$\mu_1 = \mu_2 = \mu_3$$

Thus, if it is rejected, all the ANOVA tells you is that there is a difference. We use CONTRASTS or POST HOC comparisons to determine which groups actually differ.

# How They Differ

- Post Hoc comparisons compare groups
  - Contrasts partition the variance in the model
- 
- Contrasts are used to test hypotheses
  - Post Hoc tests are used when you do not have a hypothesis

# Posthoc Comparisons

# Basic Logic

So why not just run a series of t-tests?

$$\mu_1 \text{ vs } \mu_2, \mu_2 = \mu_3, \mu_1 \text{ vs } \mu_3$$

Because we have to talk about the “family wise error rate” because we are making a series of comparisons.

# I've heard that I am safe with three comparisons, but not with four...

$\mu_1 = \mu_2 = \mu_3$  we only have one case where we can make a Type I error

This is also true for

$$\mu_1 \neq \mu_2 = \mu_3$$

$$\mu_1 = \mu_2 \neq \mu_3$$

$$\mu_1 \neq \mu_2 \neq \mu_3$$

BUT...

# I've heard that I am safe with three groups, but not four...

$$\mu_1 = \mu_2 \text{ AND } \mu_3 = \mu_4$$

Here we have TWO chances to make a Type I error, thus our family wise error rate is actually 0.1, not 0.05

Thus, we should adjust for the INFLATION of Type I error.

# Fisher's Least Significant Difference Test

$$t = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{MS_{error} \left( \frac{1}{n_1} + \frac{1}{n_2} \right)}}$$

Also known as a protected t-test

(we have replaced the estimate of pooled variance with  $MS_{error}$ )

But, this is in reality still just a t-test so it is very liberal (i.e., not your best choice).

# Other Common Posthoc Tests

Bonferroni: Essentially adjusts alpha to correct for multiple comparisons and then t-tests are used.

$$\alpha_{\text{new}} = \alpha / \text{number of tests}$$

# Other Common Posthoc Tests

Sidak's: Similar to Bonferroni but is more conservative (harder to find differences)

# Other Common Posthoc Tests

Tukey's: Reflects an adjustment to the actual formulae used to generate the  $t$  statistic, and corrects for multiple comparisons this way.

For a large number of comparisons, Tukey's Test is more powerful than Bonferroni. For a small number of comparisons, vice versa.

# Other Common Posthoc Tests

Hochberg's GT2 and Gabriel's Pairwise Comparisons Tests are similar to Tukey's Test.

Hochberg's Tests is generally more liberal.

Gabriel's Pairwise Tests are good when cell sizes are unequal.

# Other Common Posthoc Tests

Dunnett's Pairwise Tests compare all of the group means against a control mean (the first of the last). Mostly used when comparing clinical populations to a control group.

# Other Common Posthoc Tests

Tamhane's T2, Dunnett's T3, Games-Howell, and Dunnett's C are all tests to be used when the variances are unequal.

# Other Common Posthoc Tests

Waller-Duncan Test: A Bayesian (probability) approach.

Scheffe Test: Generally the most conservative, examines all linear combinations of variables.

# Simple Effects Analysis

Simple Effects Analysis denotes only using t-tests, but using a logical comparison. IE, look at the plot and do not compare things that do not make sense. If Group 2 > Group 1, and Group 3 > Group 2, why compare Group 1 and Group 3?

# Complex Procedures

Permutation Tests

FDR Methods

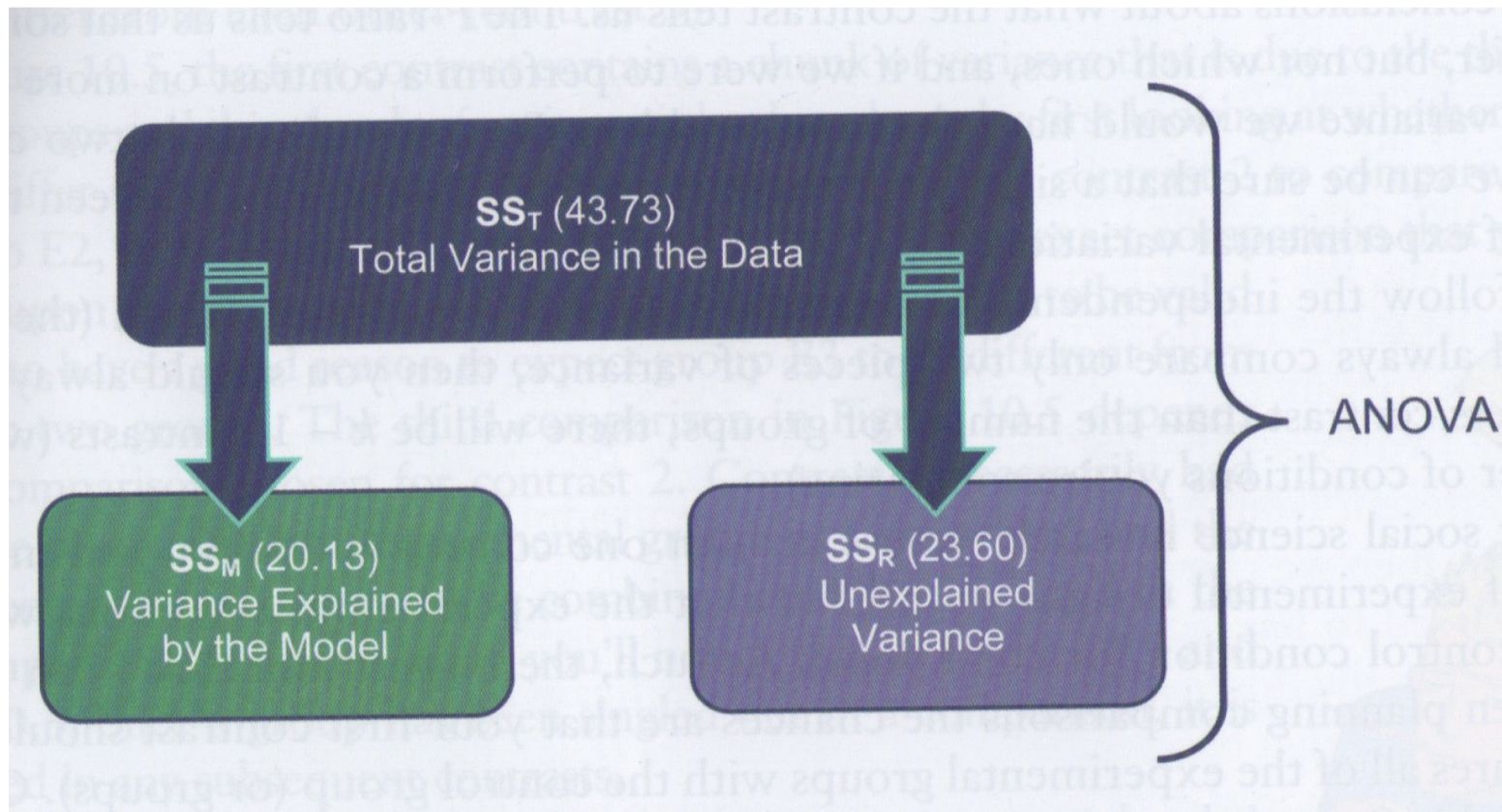
Benjamini Algorithms

# So what do I do...

1. Decide if you are going to use a contrast approach or a post hoc approach.
2. If you use a contrast approach, the key concept to get is that your contrasts **MUST** be logically defined.
3. If you use a post hoc approach, the key concept to get is that if you are making many comparisons (more than 3) you need to do something, anything really, to adjust for this.

# Contrasts

Essentially contrasts break down the variance accounted for by the model to see where the differences lie.



**$SS_M$  (20.13)**  
Variance Explained by the Model

**Low + High Dose**  
Variance Explained by  
Experimental Groups

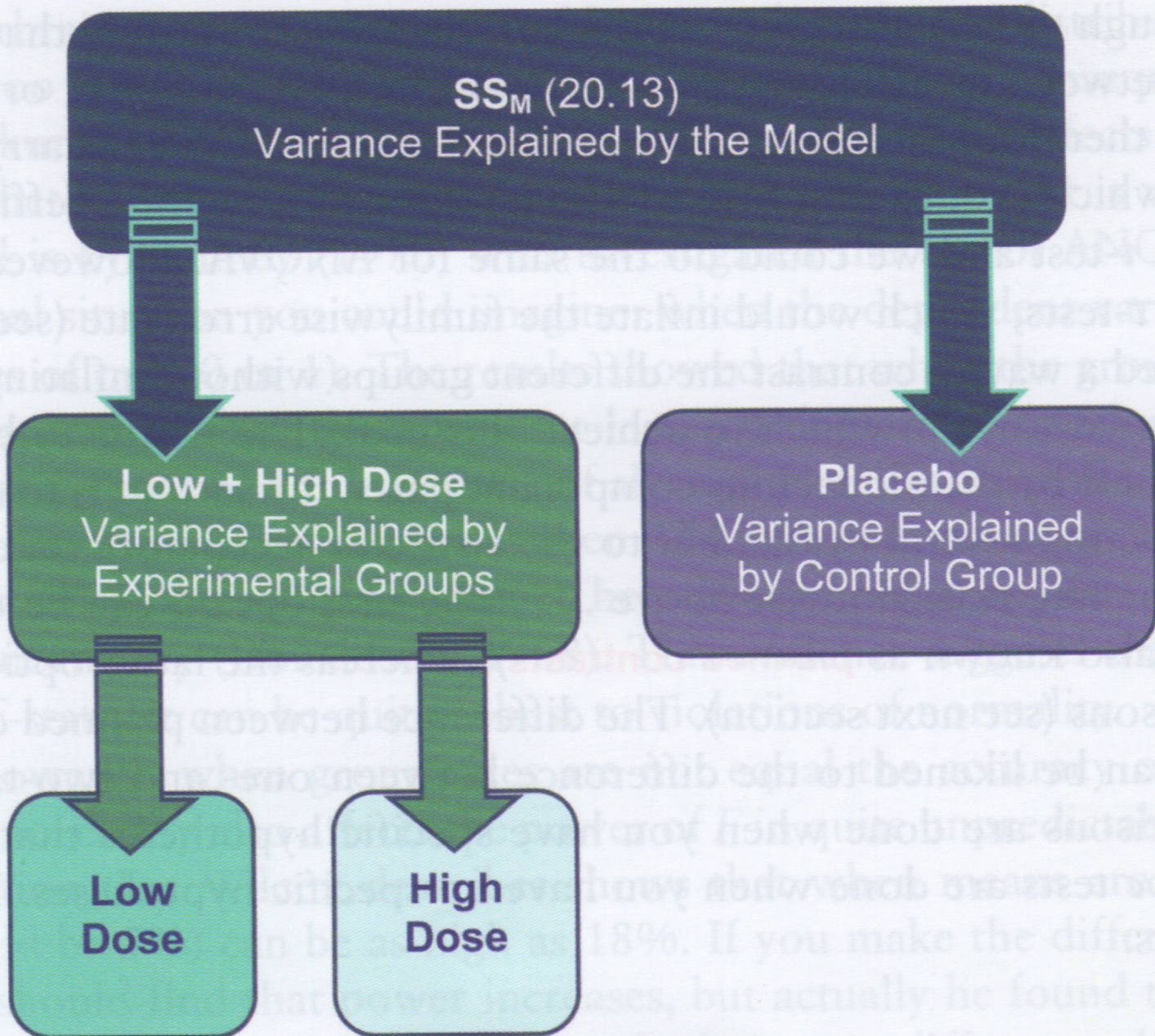
**Placebo**  
Variance Explained  
by Control Group

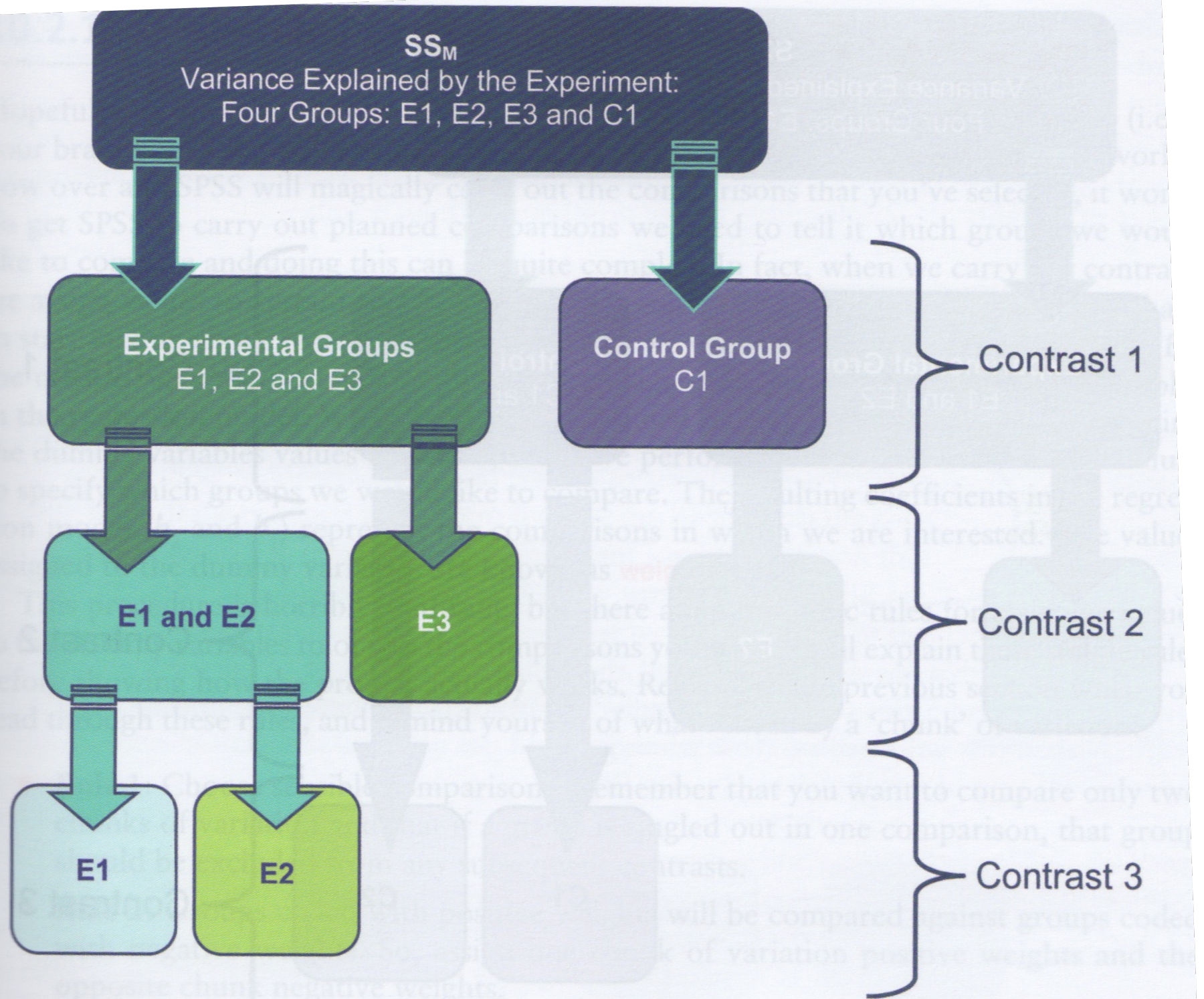
**Contrast 1**

**Low  
Dose**

**High  
Dose**

**Contrast 2**





# Rules When Using Contrasts

1. Choose sensible comparisons, you only want to compare two “chunks” of variance with each comparison. Further, if you single out a single group in a comparison, only use it in one comparison and no subsequent comparisons.

# Rules When Using Contrasts

2. Weights are used to set up contrasts. If asked, you must define the weights to sum to zero.

Contrast Group A:      +1

Contrast Group B:      - 1

# Rules When Using Contrasts

3. Groups not in a contrast should be given a weight of 0. Why?  $0 \times \text{Group Variance} = 0$  so it is negated from the contrast.

# Types of Contrasts

Deviation:

Deviation from overall mean with 1 group omitted.  
(generally not used)

Simple:

Level 2 vs Level 1, Level 3 vs Level 1, etc... (or last)

Difference:

Level 2 vs Level 1, Level 3 vs Previous, etc...

Helmert:

Level 1 vs Later, Level 2 vs Later, etc...

Repeated:

Contrasts 1 to 2, 2 to 3, 3 to 4, etc...

Polynomial:

Looks for a linear, quadratic, or cubic trend.