EPHE 357

Introduction to Research

NHST and T Tests

Olav Krigolson, PhD
krigolson@uvic.ca

Admin

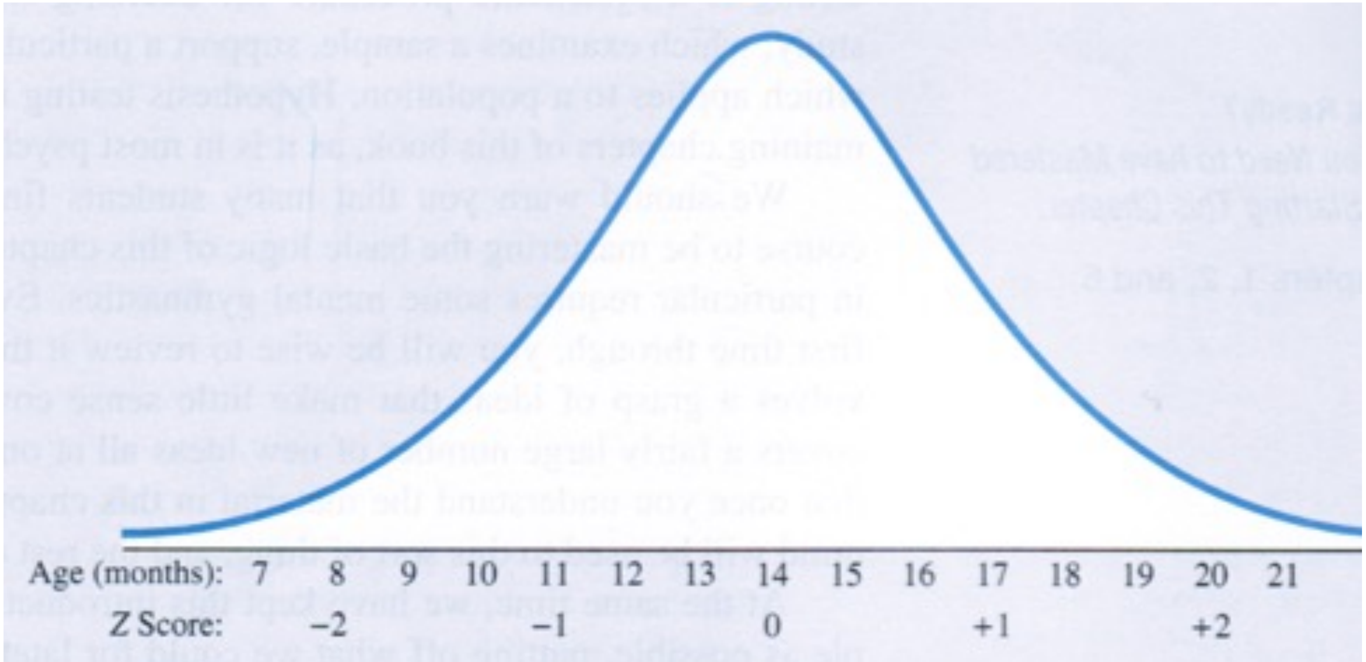
Install JASP – free stats software for next tomorrow!

Research projects – keep pushing – no set dates but this is dangerous.

Email assignments!

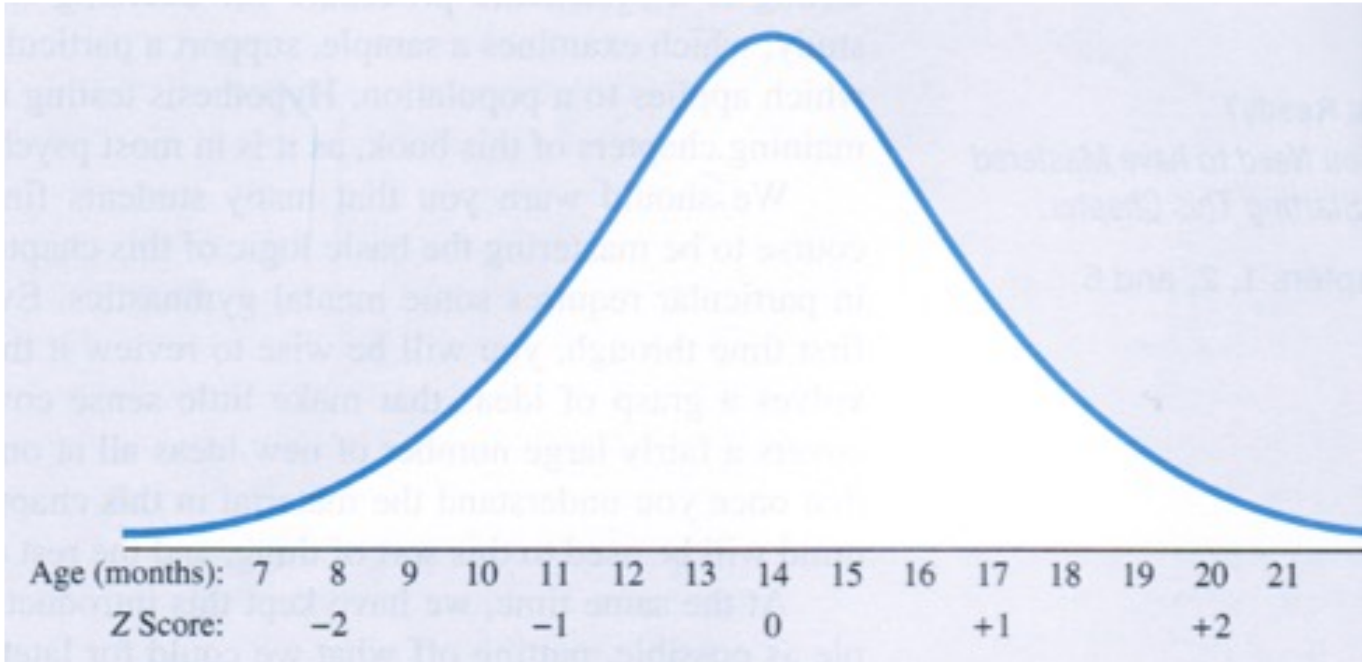
The Logic of Hypothesis Testing

How do we know if our data differs from a population?



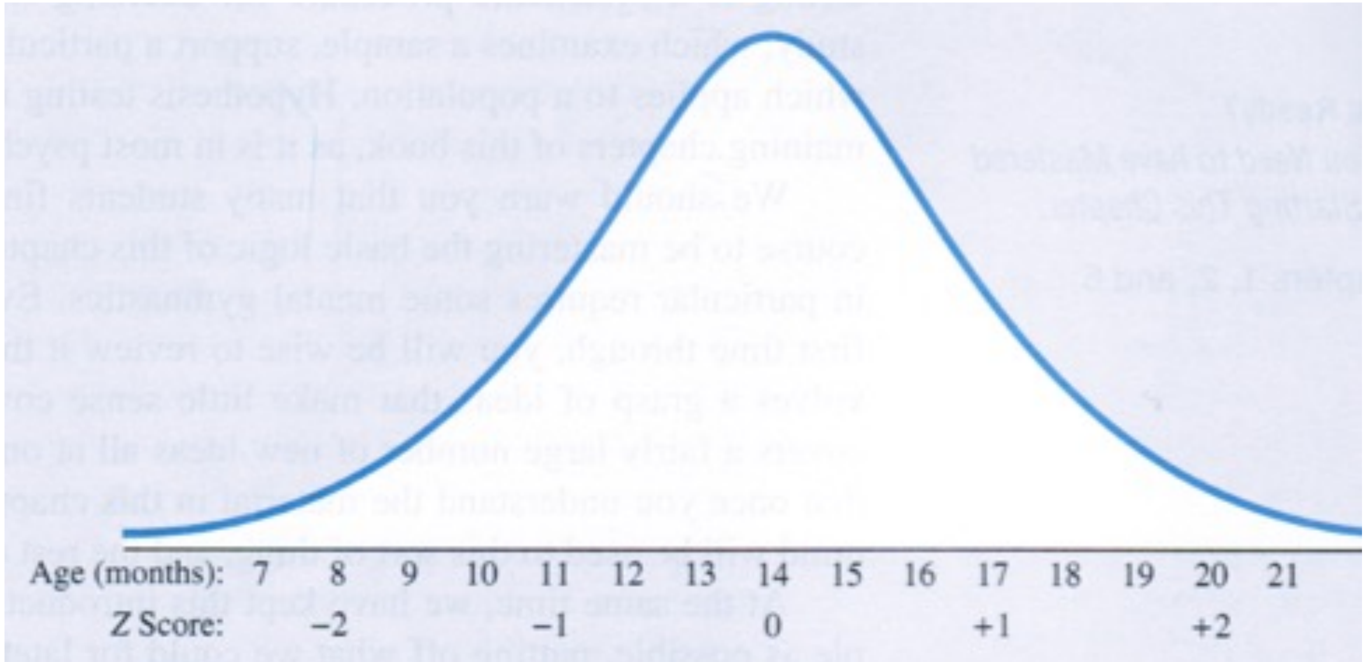
Let’s suppose the mean age for babies walking is 14 months.

We are going to try an experimental intervention that seeks to reduce that age.



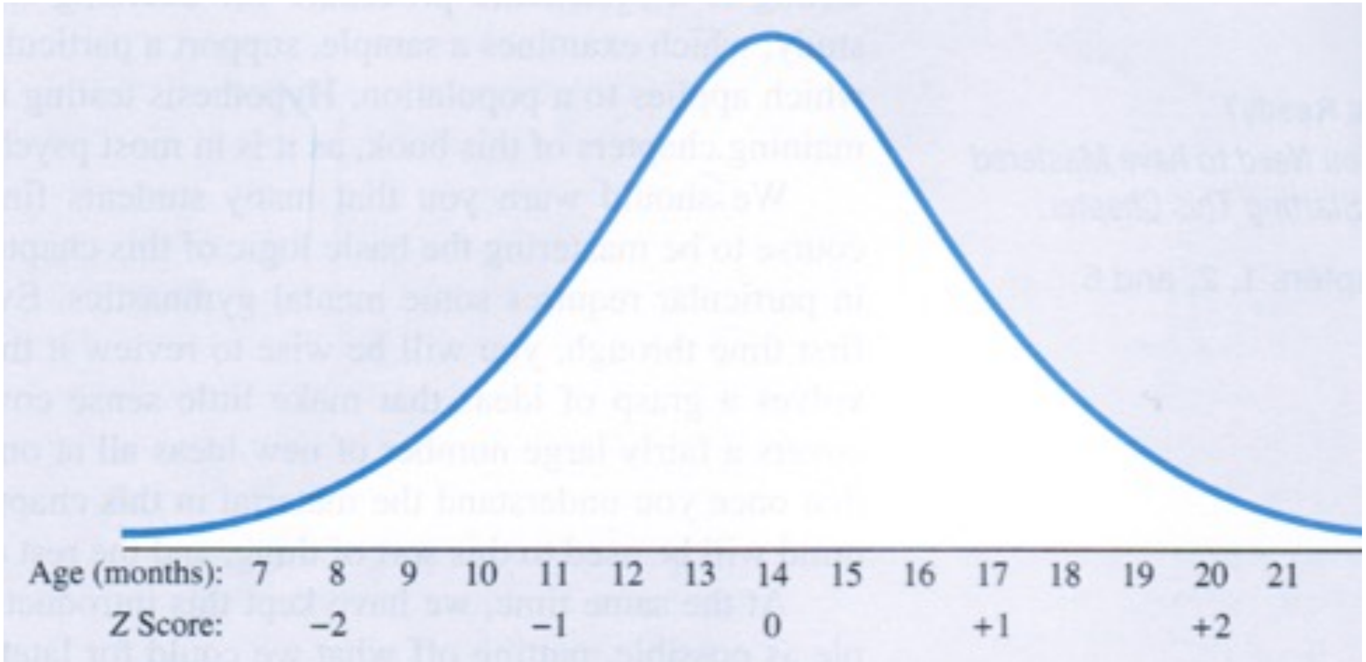
So we generate two hypotheses:

1) Our treatment does not work
2) Our treatment works



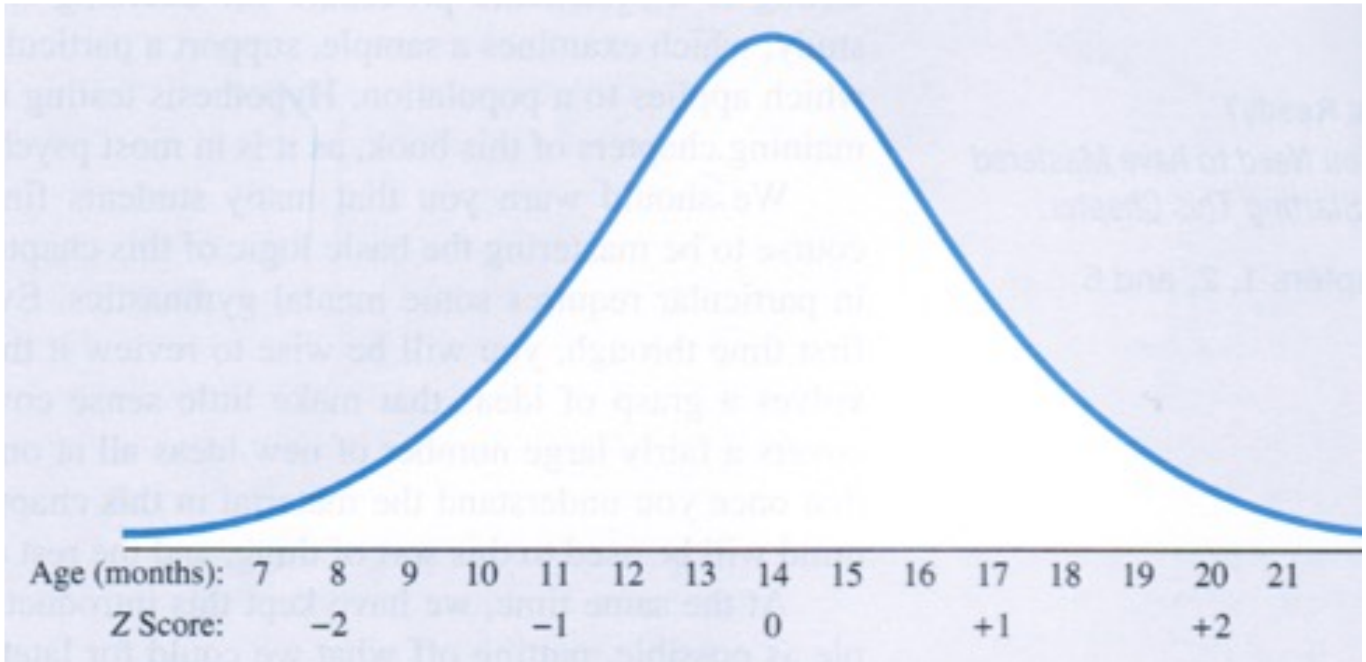
Lets think of this now in terms of populations:

Population One, babies who did not get our treatment
Population Two, babies who did get our treatment



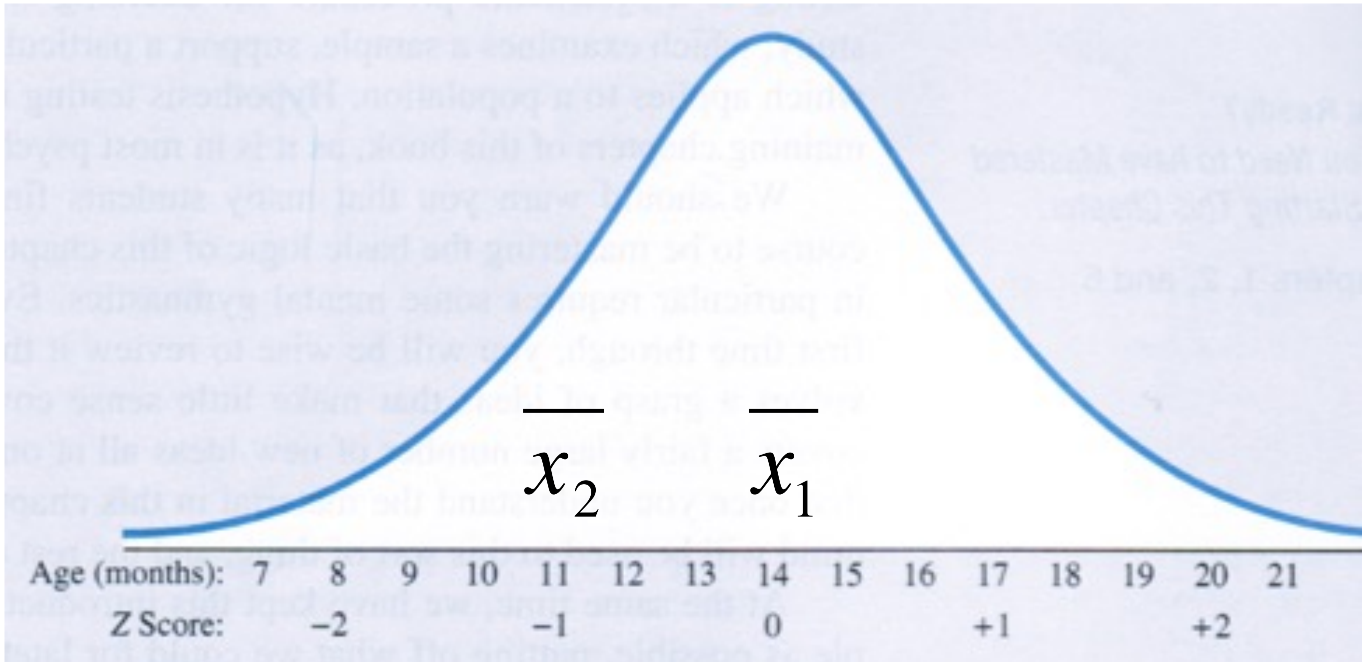
And we can quantify a descriptive statistic that is representative of our population, the mean age at which they begin walking

Population One: mean age of walking: μ_1
Population Two: mean age of walking: μ_2



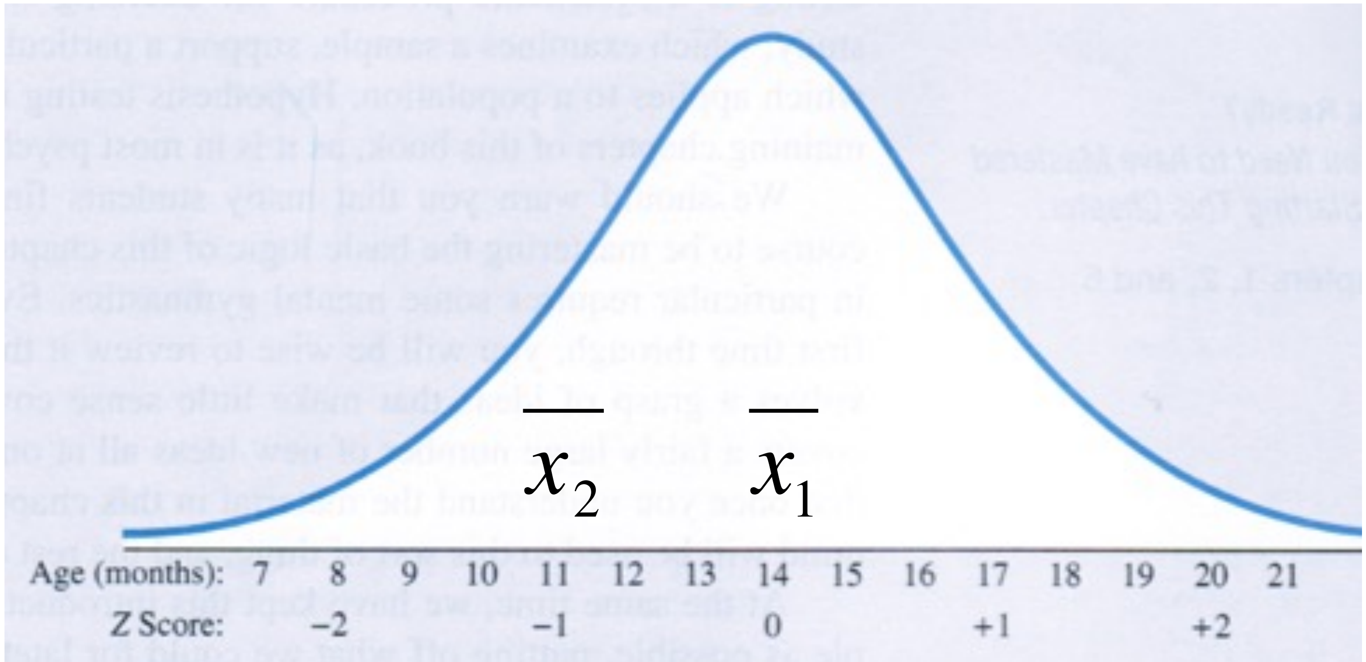
We typically frame this within the context of the null and alternative hypotheses:

1) H_0: The Null Hypothesis: Our treatment does not work: $\mu_1 = \mu_2$
2) H_1: The Alternative Hypothesis: Our treatment works: $\mu_1 \neq \mu_2$



So, let's say we run the study, and we obtain the following data.

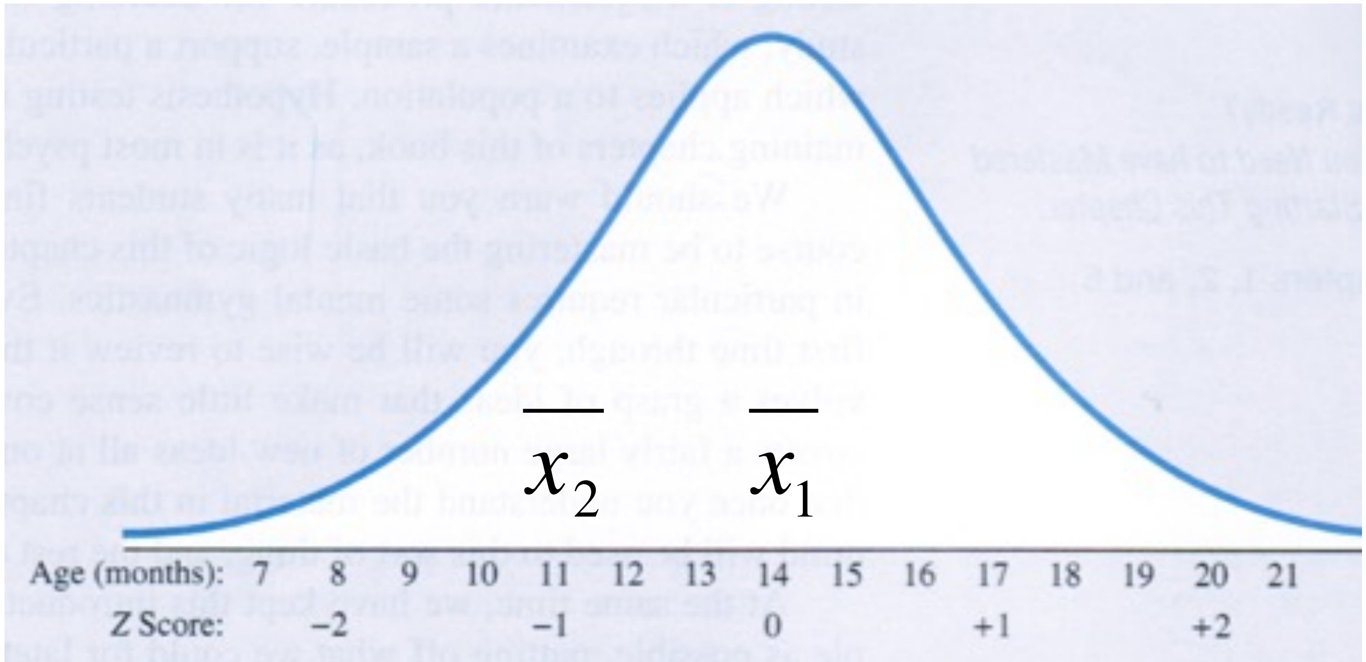
How do we know if our treatment worked?



Here’s the weird bit, we first want to find out the probability of getting our result if the null hypothesis is true.

We call this our “critical value”

The probability of our results at which we will discard the null hypothesis and accept the alternative hypothesis.



0.05

The point at which a sample score is so extreme that we discard the null hypothesis

To do this we need to know the sampling distribution to perform statistical tests

The sampling distribution of the mean

provides all of the values the mean can take, along with the probability of getting each value if sampling is random from the null-hypothesis population

Consider the following…

Imagine you have a population that consists of 5 scores:

2, 3, 4, 5, 6

If you take samples of size 2, how many different possible samples are there and what do they consist of?

Sample		Sample	
1	2,2	14	4,5
2	2,3	15	4,6
3	2,4	16	5,2
4	2,5	17	5,3
5	2,6	18	5,4
6	3,2	19	5,5
7	3,3	20	5,6
8	3,4	21	6,2
9	3,5	22	6,3
10	3,6	23	6,4
11	4,2	24	6,5
12	4,3	25	6,6
13	4,4		

The sampling distribution of the mean

provides all of the values the mean can take, along with the probability of getting each value if sampling is random from the null-hypothesis population

Sample		$\bar{x}$	Sample		$\bar{x}$
1	2,2	2	14	4,5	4.5
2	2,3	2.5	15	4,6	5
3	2,4	3	16	5,2	3.5
4	2,5	3.5	17	5,3	4
5	2,6	4	18	5,4	4.5
6	3,2	2.5	19	5,5	5
7	3,3	3	20	5,6	5.5
8	3,4	3.5	21	6,2	4
9	3,5	4	22	6,3	4.5
10	3,6	4.5	23	6,4	5
11	4,2	3	24	6,5	5.5
12	4,3	3.5	25	6,6	6
13	4,4	4			

The sampling distribution of the mean

provides all of the values the mean can take, **along with the probability of getting each value if sampling is random from the null-hypothesis population**

Sample		$\bar{x}$	Sample		$\bar{x}$
$\boxed{1}$	$\boxed{2,2}$	$\boxed{2}$	14	4,5	4.5
2	2,3	2.5	15	4,6	5
3	2,4	3	16	5,2	3.5
4	2,5	3.5	17	5,3	4
5	2,6	4	18	5,4	4.5
6	3,2	2.5	19	5,5	5
7	3,3	3	20	5,6	5.5
8	3,4	3.5	21	6,2	4
9	3,5	4	22	6,3	4.5
10	3,6	4.5	23	6,4	5
11	4,2	3	24	6,5	5.5
12	4,3	3.5	25	6,6	6
13	4,4	4			

Probability of getting the mean

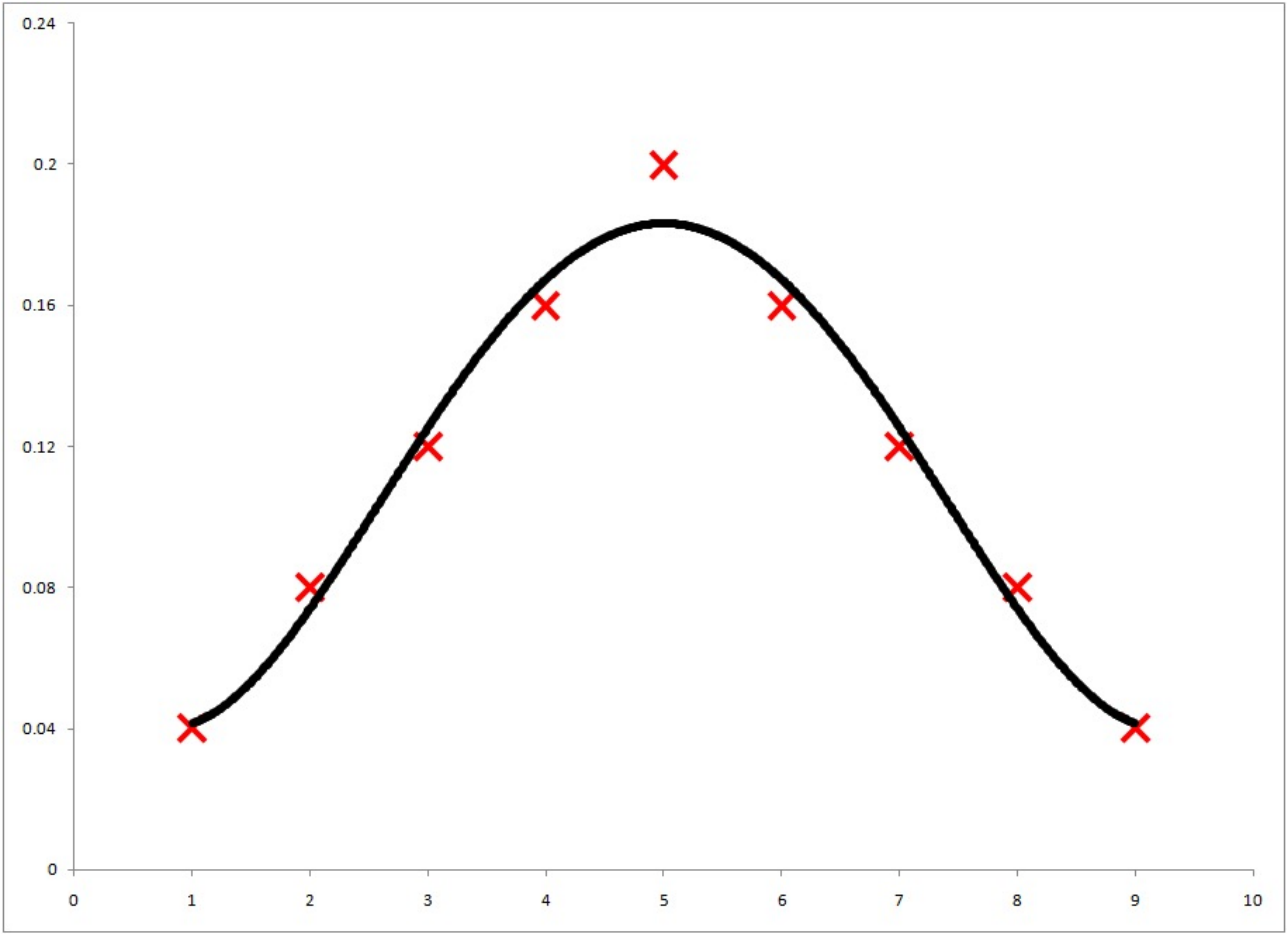
p ($\bar{x}$) = 2.0 1/25 0.04

Probability of getting the mean

p ($\bar{x}$) = 2.0	1/25	0.04
p ($\bar{x}$) = 2.5	2/25	0.08
p ($\bar{x}$) = 3.0	3/25	0.12
p ($\bar{x}$) = 3.5	4/25	0.16
p ($\bar{x}$) = 4.0	5/25	0.20
p ($\bar{x}$) = 4.5	4/25	0.16
p ($\bar{x}$) = 5.0	3/25	0.12
p ($\bar{x}$) = 5.5	2/25	0.08
p ($\bar{x}$) = 6.0	1/25	0.04

Sampling distribution of the mean

$\overline{x}$	p($\overline{x}$)
2	0.04
2.5	0.08
3	0.12
3.5	0.16
4	0.20
4.5	0.16
5	0.12
5.5	0.08
6	0.04



0.24
0.2
0.16
0.12
0.08
0.04
0
0
1
2
3
4
5
6
7
8
9
10

Characteristics of the Sampling Distribution of the Mean

The sampling distribution of the mean is normal distriubtion

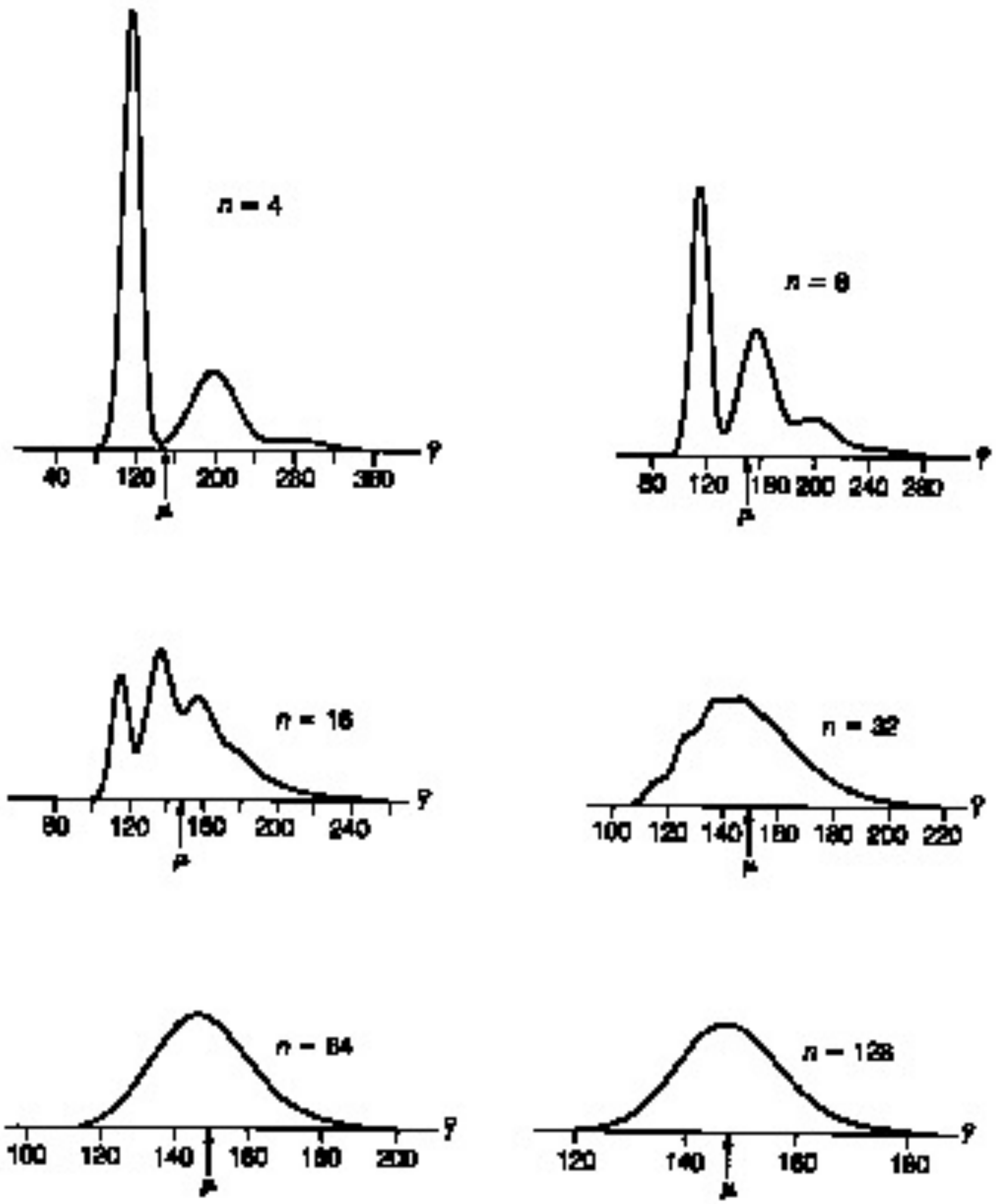
Characteristics of the Sampling Distribution of the Mean

The Central Limit Theorem

REGARDLESS of the shape of the population of raw scores, the sampling distribution of the mean approaches a normal distribution as sample size N increases

[illegible]

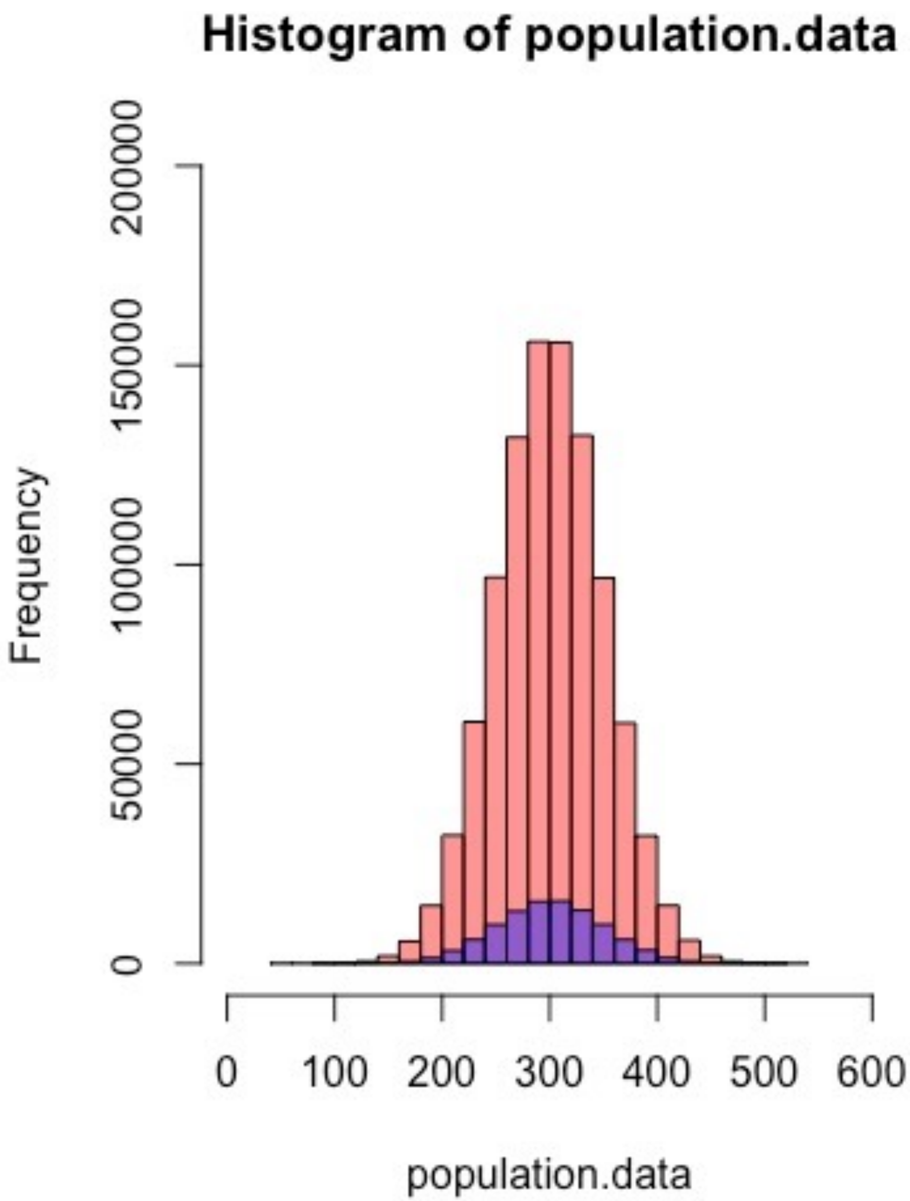
Sampling Distributions of $\bar{y}$ for Samples from the Two-Store Population

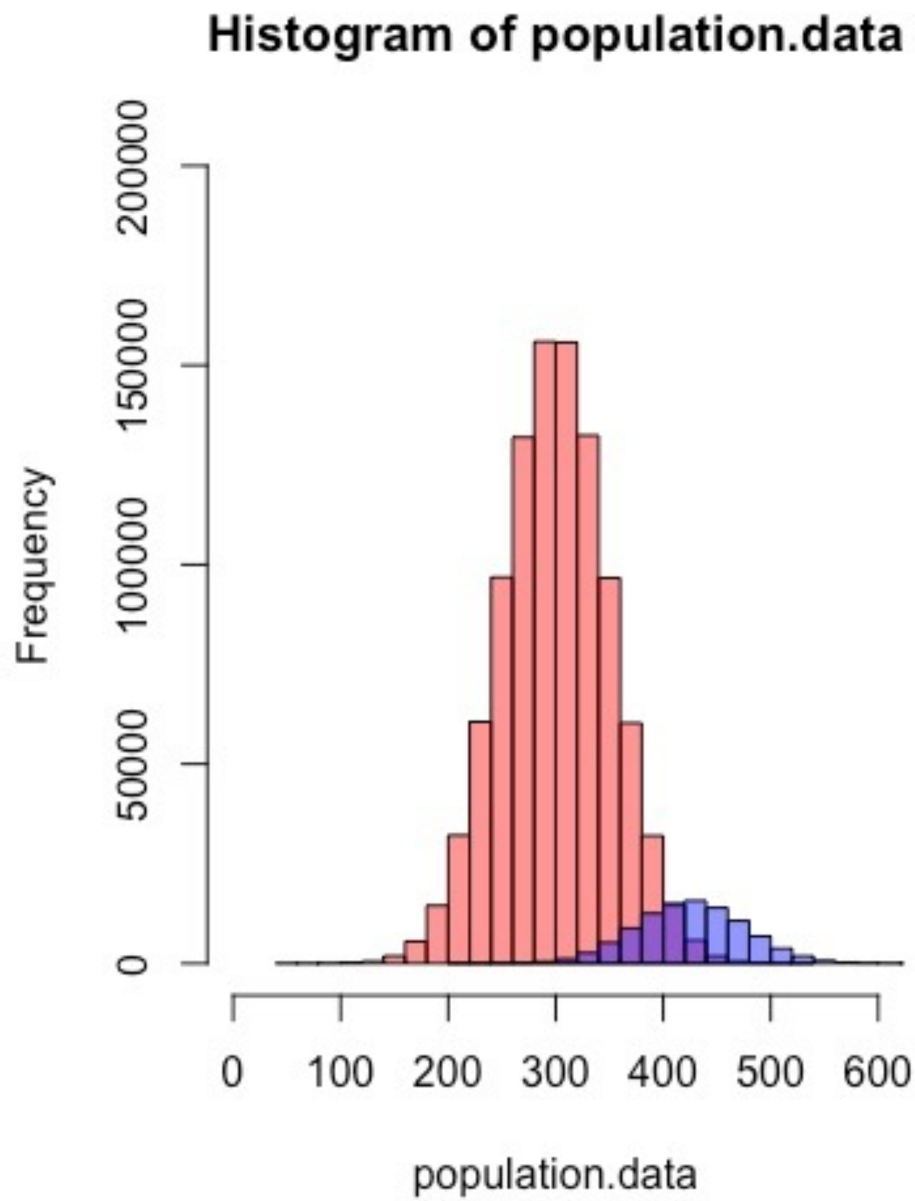


The Problem…

There is no way to know the sampling distribution of the mean for any data that we collect… so what do we compare our sample mean to???

What do we do?


Histogram of population.data
Frequency
0
50000
100000
150000
200000
0
100
200
300
400
500
600
population.data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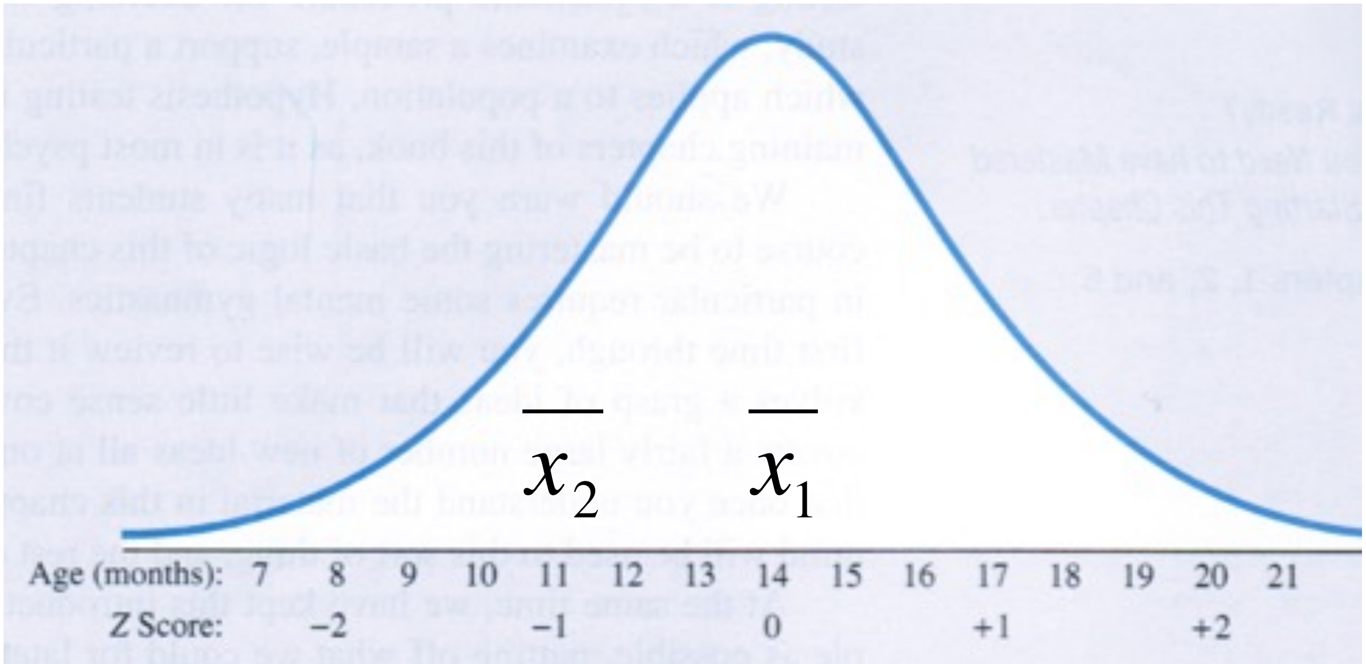

We could just use the normal distribution…

WHY?

1. We know that the sampling distribution of the mean for any variable is normal given a reasonable sample size

2. We know the values and probabilities of the normal distribution

Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9924	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9958	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986

We could compare the data against a population with mean of 0 and standard deviation of 1 – the normal distribution… by doing this, we get the probability that the mean of our data comes from the normal distribution. But…

But…

The normal distribution is dead to use unless we know the standard deviation of the population…

$$z = \frac{\text{sample mean} - \text{population mean}}{\text{population standard deviation}}$$

So what now?

What do we do?

We compute some statistics from our data for which there is a known probability of scores given a specific sample size.

t = sample mean – population mean

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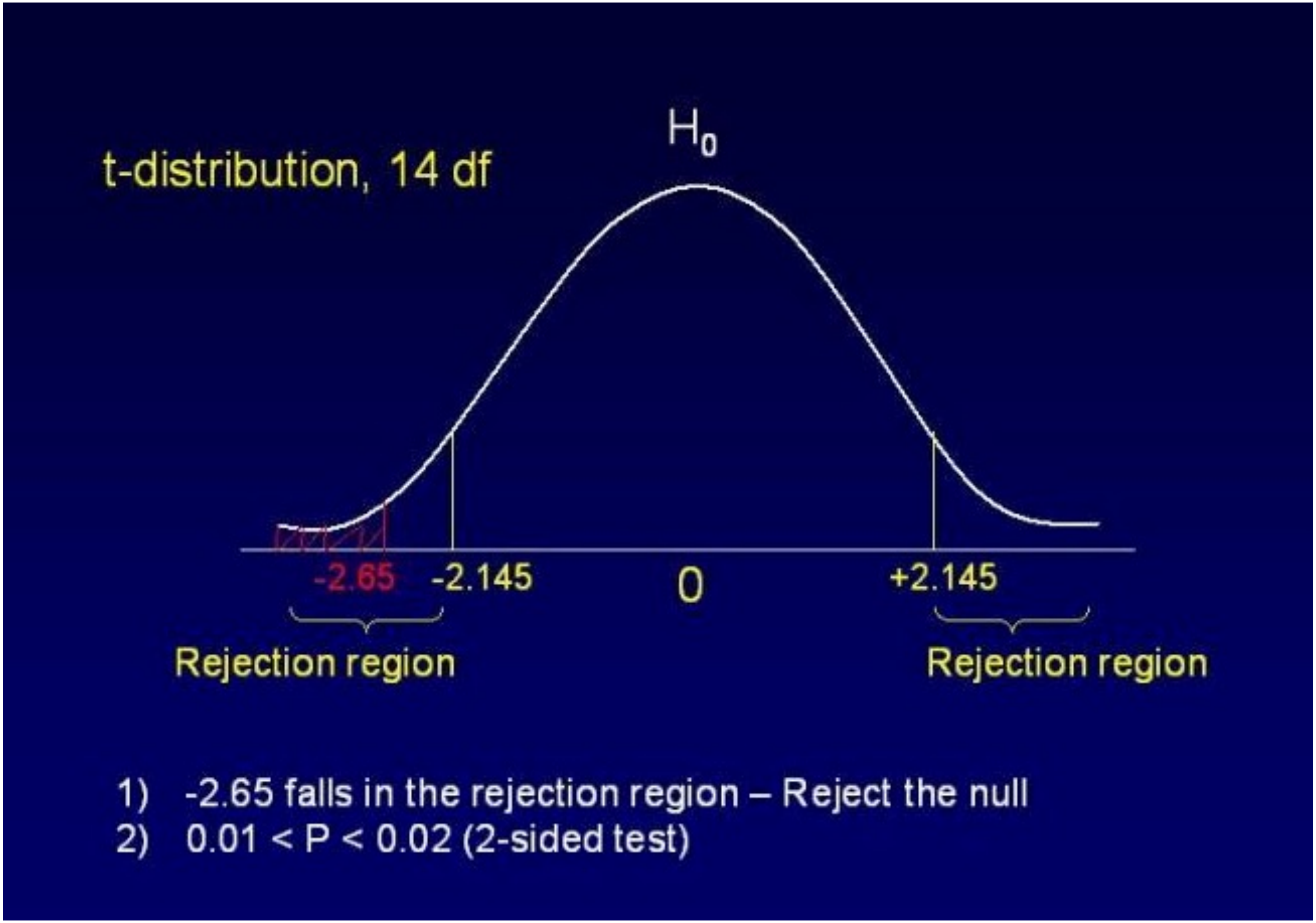
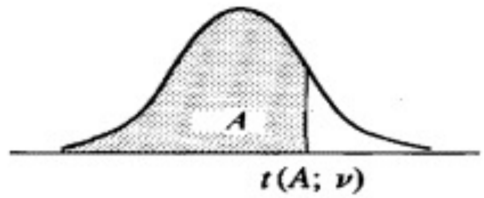
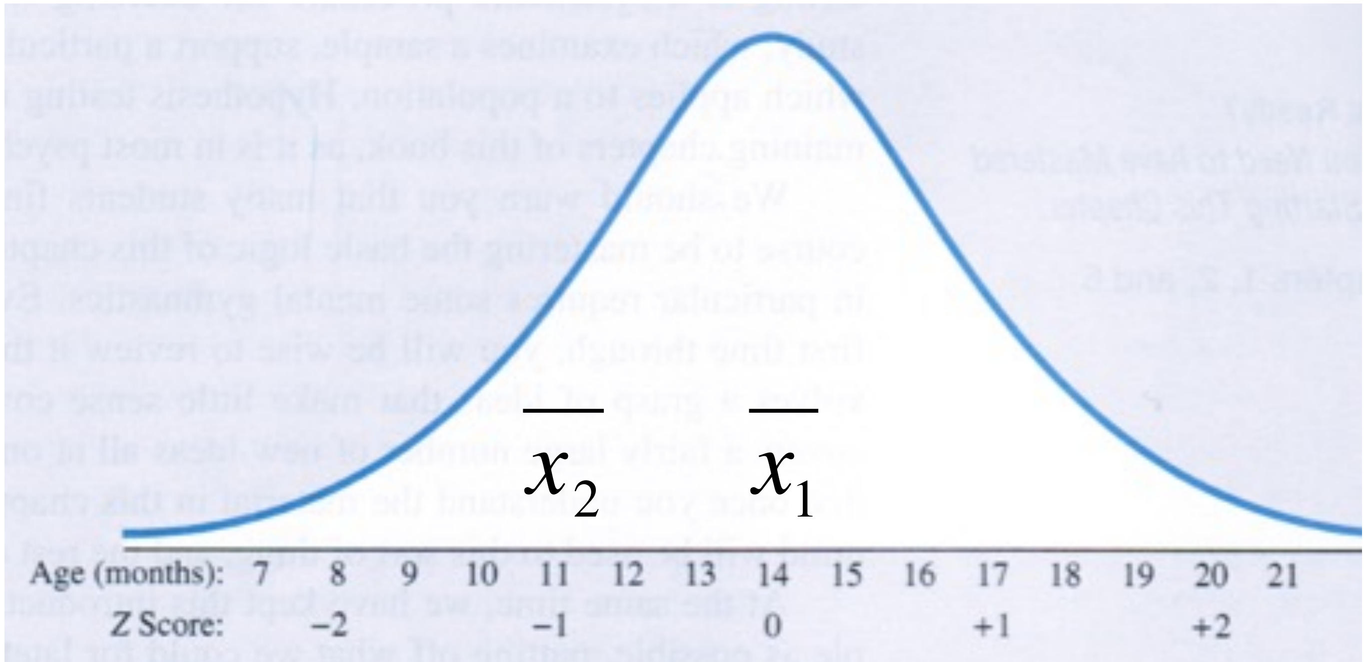
standard deviation of sample


t-distribution, 14 df
H0
-2.65
-2.145
0
+2.145
Rejection region
Rejection region
1) -2.65 falls in the rejection region – Reject the null
2) 0.01 < P < 0.02 (2-sided test)

Entry is $t(A; \nu)$ where $P\{t(\nu) \leq t(A; \nu)\} = A$



ν	.60	.70	.80	.85	.90	.95	.975
1	0.325	0.727	1.376	1.963	3.078	6.314	12.706
2	0.289	0.617	1.061	1.386	1.886	2.920	4.303
3	0.277	0.584	0.978	1.250	1.638	2.353	3.182
4	0.271	0.569	0.941	1.190	1.533	2.132	2.776
5	0.267	0.559	0.920	1.156	1.476	2.015	2.571
6	0.265	0.553	0.906	1.134	1.440	1.943	2.447
7	0.263	0.549	0.896	1.119	1.415	1.895	2.365
8	0.262	0.546	0.889	1.108	1.397	1.860	2.306
9	0.261	0.543	0.883	1.100	1.383	1.833	2.262
10	0.260	0.542	0.879	1.093	1.372	1.812	2.228
11	0.260	0.540	0.876	1.088	1.363	1.796	2.201
12	0.259	0.539	0.873	1.083	1.356	1.782	2.179
13	0.259	0.537	0.870	1.079	1.350	1.771	2.160
14	0.258	0.537	0.868	1.076	1.345	1.761	2.145
15	0.258	0.536	0.866	1.074	1.341	1.753	2.131
16	0.258	0.535	0.865	1.071	1.337	1.746	2.120
17	0.257	0.534	0.863	1.069	1.333	1.740	2.110
18	0.257	0.534	0.862	1.067	1.330	1.734	2.101
19	0.257	0.533	0.861	1.066	1.328	1.729	2.093
20	0.257	0.533	0.860	1.064	1.325	1.725	2.086
21	0.257	0.532	0.859	1.063	1.323	1.721	2.080
22	0.256	0.532	0.858	1.061	1.321	1.717	2.074
23	0.256	0.532	0.858	1.060	1.319	1.714	2.069
24	0.256	0.531	0.857	1.059	1.318	1.711	2.064
25	0.256	0.531	0.856	1.058	1.316	1.708	2.060
26	0.256	0.531	0.856	1.058	1.315	1.706	2.056
27	0.256	0.531	0.855	1.057	1.314	1.703	2.052
28	0.256	0.530	0.855	1.056	1.313	1.701	2.048
29	0.256	0.530	0.854	1.055	1.311	1.699	2.045
30	0.256	0.530	0.854	1.055	1.310	1.697	2.042
40	0.255	0.529	0.851	1.050	1.303	1.684	2.021
60	0.254	0.527	0.848	1.045	1.296	1.671	2.000
120	0.254	0.526	0.845	1.041	1.289	1.658	1.980
∞	0.253	0.524	0.842	1.036	1.282	1.645	1.960



So what are we doing with z scores, t scores, F ratios, etc…

We are computing a statistic which is our samples score on the sampling distribution of the null hypothesis…

And WE ASSUME these magical distributions approximate the sampling distribution of the mean of our data…

As stated, to draw these conclusions we need a test statistic to evaluation against the appropriate sampling distribution.

$$\text{Test Statistic} = \frac{\text{variance explained by model}}{\text{variance not explained by model}}$$

$$\text{Test Statistic} = \frac{\text{effect}}{\text{error}}$$

And then we can calculate the probability of getting the test statistic we have obtained and evaluating it against alpha.

alpha = 0.05

if $p < 0.05$ then we say our test statistic is different

If $p > 0.05$ then we say our test statistic is the same

Type I and Type II Errors

<u>Type I Error</u>

Reject the null hypothesis when it is true.

<u>Type II Error</u>

Retain the null hypothesis when it is false.

Controlling for the possibility of Type I and Type II errors is not easy...

If we reduce alpha to reduce the chance of a Type I error, we increase the likelihood that we are making a Type II error!

Almost everything you wanted to know about t-tests

Three flavours

1. Single Sample
2. Dependent Samples
3. Independent Samples

Recall the almighty z score

$$z = \frac{\bar{x} - u}{\sigma_{\bar{x}}}$$

Note, this is a VALID test if you know the POPULATION mean and the POPULATION standard deviation!

Single Sample TTest

You want to compare a sample to a known population mean but you DO NOT know the population standard deviation.

Revisiting the logic of hypothesis testing…

1) <u>Check assumptions</u>

2) Calculating an appropriate statistic (e.g., the number of plus and minuses

3) Evaluating that statistic relative to the appropriate sampling distribution

Assumptions

1. Data are continuous
2. Data are independent
3. No outliers
4. Normality

Revisiting the logic of hypothesis testing…

1) Check assumptions

2) <u>Calculating an appropriate statistic (e.g., t)</u>

3) Evaluating that statistic relative to the appropriate sampling distribution

The t-test for Single Samples

$$t = \frac{\bar{x} - u}{s_{\bar{x}}}$$

The t-test allows comparison between a sample mean and a population mean, given that we know the **sample standard deviation.**

Revisiting the logic of hypothesis testing…

1) Check assumptions

2) Calculating an appropriate statistic (e.g., the number of plus and minuses

3) <u>Evaluating that statistic relative to the appropriate sampling distribution</u>

Sampling Distribution of t

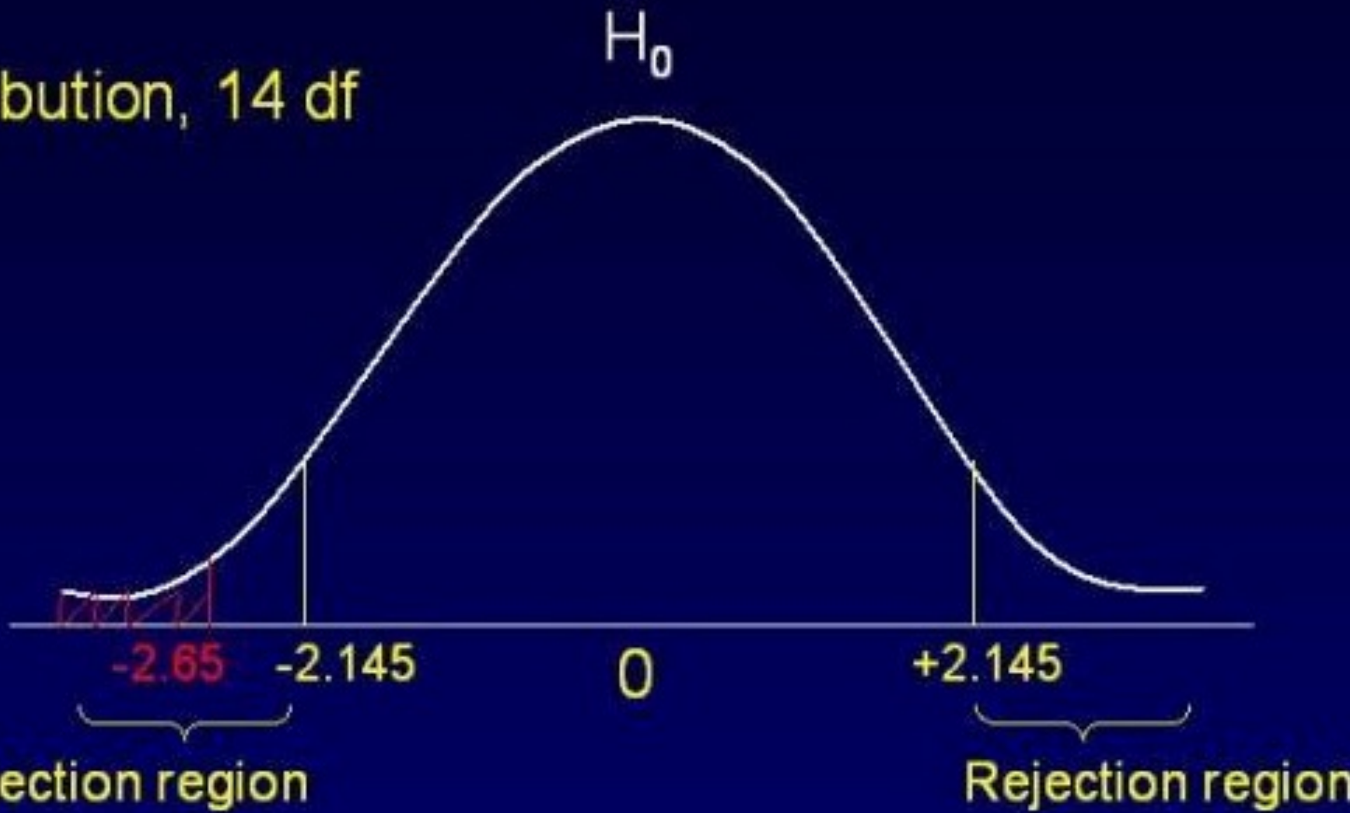
Defn:

A probability distribution of t-values that would occur if all possible different samples of a fixed size N were drawn from the null-hypothesis population.

Provides:

All possible different t-values for sample size N

The probability of getting each value if sampling is random from the null hypothesis population t-distribution, 14 df



1) -2.65 falls in the rejection region – Reject the null
2) $0.01 < P < 0.02$ (2-sided test)

Determine t-critical…

Which is why we use computers…

Degrees of Freedom

$$df = n - 1$$

Dependent Samples T-Test
(Paired)
(Correlated Groups)

Paired Samples TTest

You want to compare two related or dependent samples.

i) all people have been measured at two different time points

ii) all people have completed two experimental conditions

Revisiting the logic of hypothesis testing…

1) Check assumptions

2) Calculating an appropriate statistic (e.g., the number of plus and minuses

3) Evaluating that statistic relative to the appropriate sampling distribution

Assumptions

1. Data are continuous
2. Data are independent
3. No outliers
4. Normality

Revisiting the logic of hypothesis testing…

1) Check assumptions

<u>2) Calculating an appropriate statistic (e.g., t)</u>

3) Evaluating that statistic relative to the appropriate sampling distribution

The t-test for Dependent Samples

$$t = \frac{\bar{x}_d - 0}{s_{\bar{x}_d}}$$

The t-test allows comparison between two sample means that are dependent upon each other

Revisiting the logic of hypothesis testing…

1) Check assumptions

2) Calculating an appropriate statistic (e.g., the number of plus and minuses

3) <u>Evaluating that statistic relative to the appropriate sampling distribution</u>

USE A COMPUTER

Independent Samples T-Test
(Between)

Independent Samples TTest

You want to compare two independent samples.

Revisiting the logic of hypothesis testing…

1) Check assumptions

2) Calculating an appropriate statistic (e.g., t)

3) Evaluating that statistic relative to the appropriate sampling distribution

Assumptions

1. Data are continuous
2. Data are independent
3. No outliers
4. Normality
5. Homogeneity of Variance

The t-test for Independent Samples

Note, that if the assumption of homogeneity of variance is broken, we have heterogeneity of variance.

Most statistical programs correct for this by adjusting the degrees of freedom.
(see Howell, 2002)

The t-test is *ROBUS*T (insensitive to these violations)

Especially if n >= 30 and n1 = n2

Revisiting the logic of hypothesis testing…

1) Check assumptions

2) <u>Calculating an appropriate statistic (e.g., t)</u>

3) Evaluating that statistic relative to the appropriate sampling distribution

The t-test for Independent Samples

$$t = \frac{\overline{x_1} - \overline{x_2}}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

The t-test allows comparison between two sample means that are dependent upon each other

$$t = \frac{\overline{x_1} - \overline{x_2}}{\sqrt{s_p^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

$$s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$$

With pooled variance

Degrees of Freedom

$$df = n1 + n2 - 2$$

Revisiting the logic of hypothesis testing…

1) Check assumptions

2) Calculating an appropriate statistic (e.g., t)

3) <u>Evaluating that statistic relative to the appropriate sampling distribution</u>

Use a computer!

Question for Understanding…

Why is homogeneity of variance not an assumption for single and paired samples t-tests?