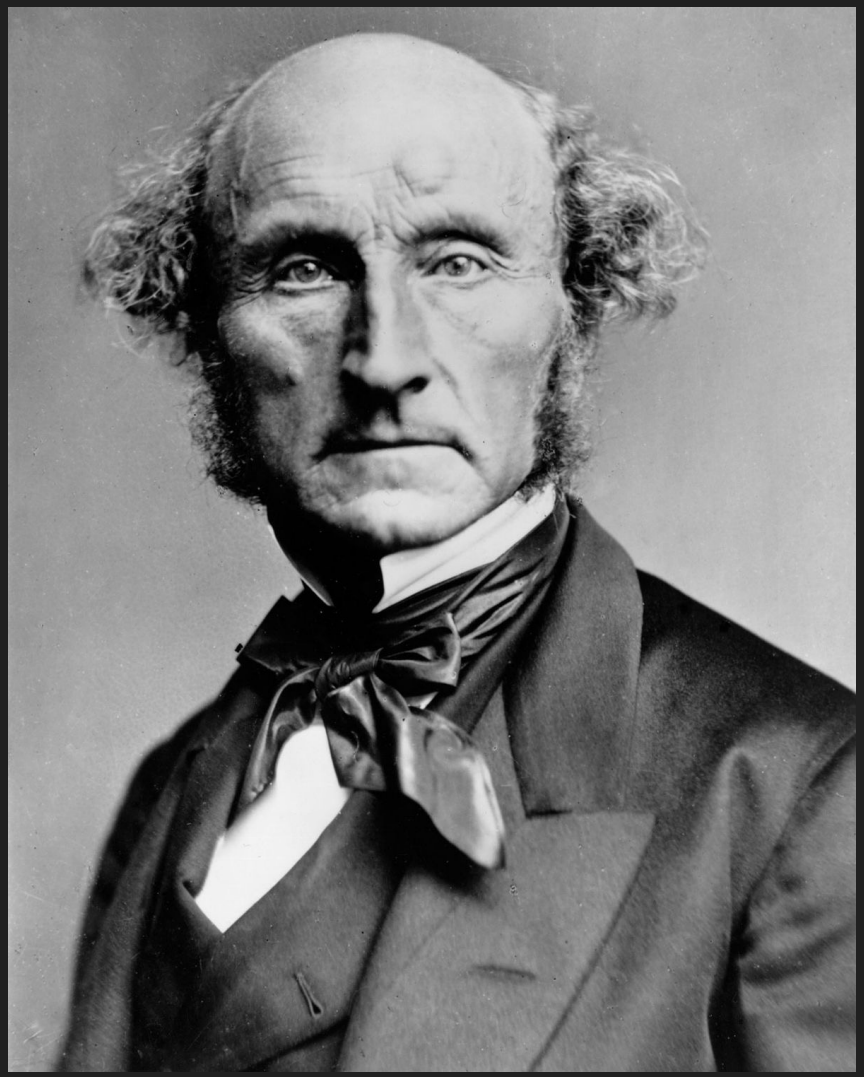
DECISION MAKING: EXPECTED VALUE

MEDS 470 / NRSC 500B

Dr. Olav E. Krigolson

Decision Making Theory



Utilitarianism

People seek actions
that increase utility
and avoid actions that
decrease utility

Mill, 1861

Decision Making

Our ability to process multiple alternatives and choose the option that maximizes utility

Huygens, 1657

Expected Value = Value x Probability

Expanded Form

$$EV = Gain \times P_G - Cost \times P_C$$

A Sample Problem

Problem 1

Would you play a gamble that has a 40% chance to win $1000 or a 70% chance to win $600?

Another Expected Value Problem

Example: the Lottery

- The Lottery (also known as a tax on people who are bad at math…)
- A certain lottery works by picking 6 numbers from 1 to 49. It costs $1.00 to play the lottery, and if you win, you win $2 million after taxes.
- *If you play the lottery once, what are your expected winnings or losses?*

Lottery

Calculate the probability of winning in 1 try:

$$\frac{1}{\binom{49}{6}} = \frac{1}{\frac{49!}{43!6!}} = \frac{1}{13{,}983{,}816} = 7.2 \times 10^{-8}$$

"49 choose 6"

Out of 49 numbers, this is the number of distinct combinations of 6.

The probability function (note, sums to 1.0):

x$	*p(x)*
-1	.999999928
+ 2 million	7.2×10^{-8}

Expected Value

The probability function

x$	*p(x)*
-1	.999999928
+ 2 million	7.2×10^{-8}

Expected Value

E(X) = P(win)*$2,000,000 + P(lose)*-$1.00

$= 2.0 \times 10^{6} * 7.2 \times 10^{-8} + .999999928\ (-1) = .144 - .999999928 = -\$.86$

Negative expected value is never good!

You shouldn't play if you expect to lose money!

Expected Value

If you play the lottery every week for 10 years, what are your expected winnings or losses?

520 x (-.86) = -$447.20

Why casinos give out free drinks

A roulette wheel has the numbers 1 through 36, as well as 0 and 00. If you bet $1 that an odd number comes up, you win or lose $1 according to whether or not that event occurs. If random variable X denotes your net gain, X=1 with probability 18/38 and X= -1 with probability 20/38.

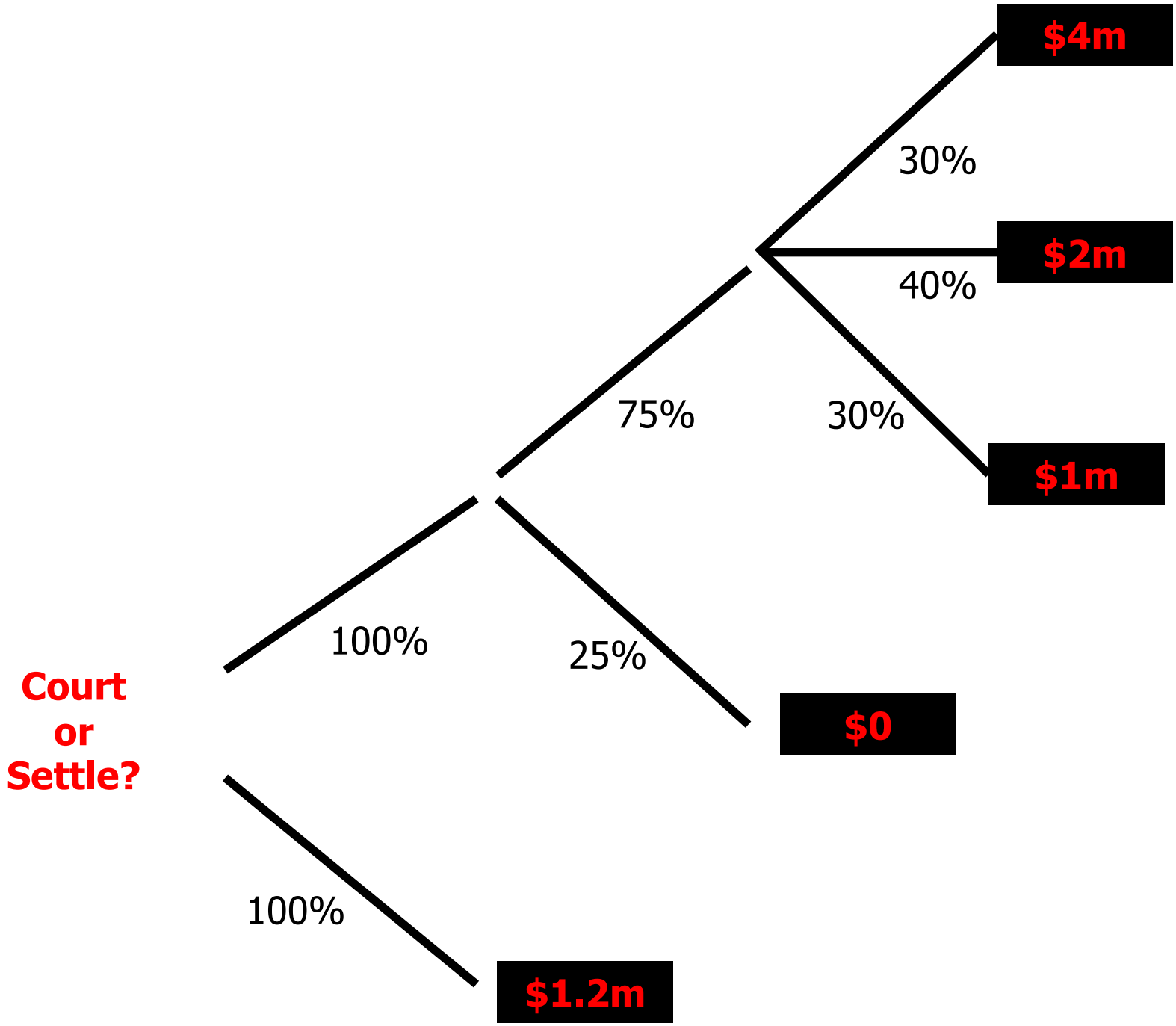
E(X) = 1(18/38) – 1 (20/38) = -$.053

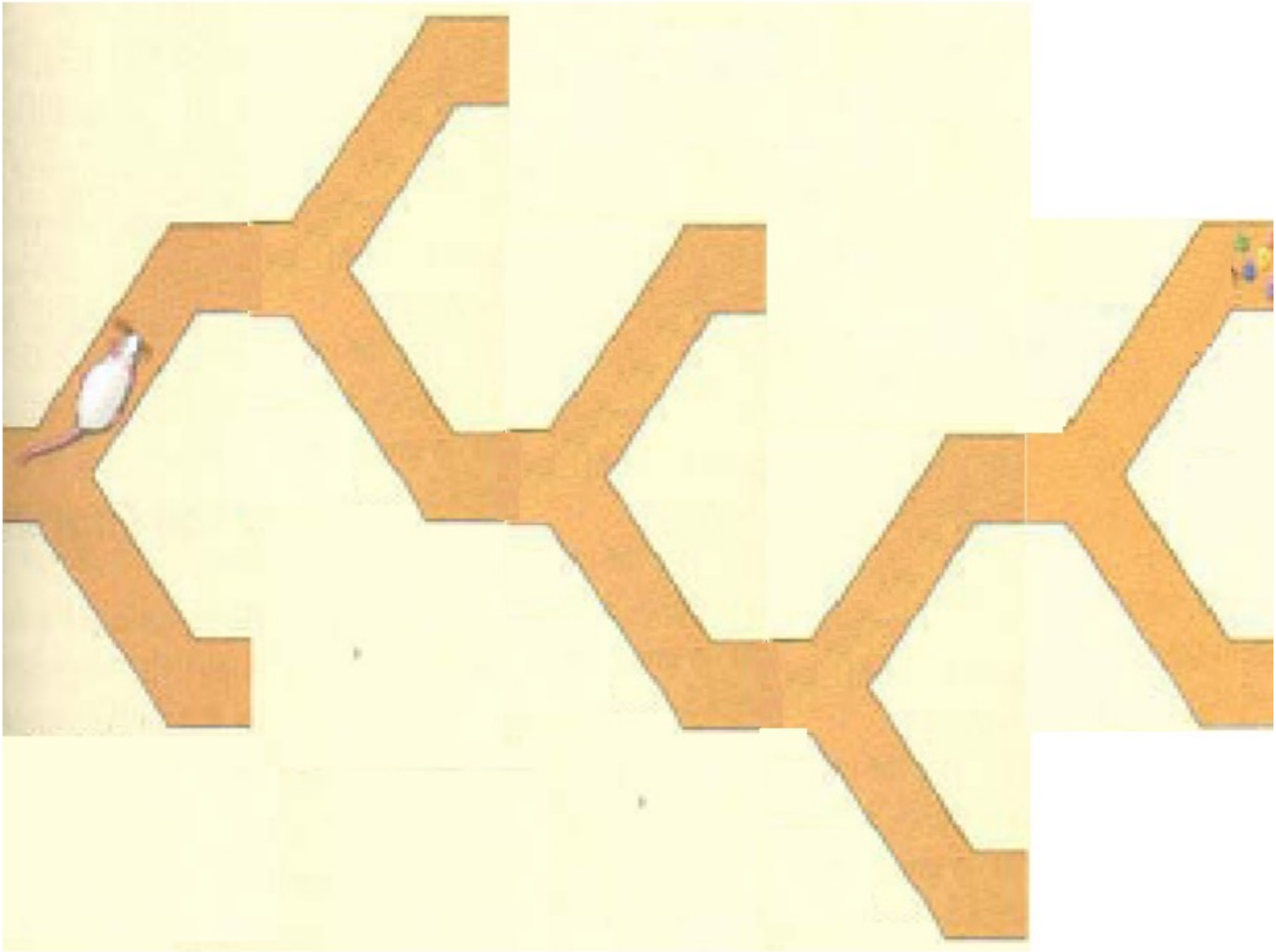
On average, the casino wins (and the player loses) 5 cents per game.

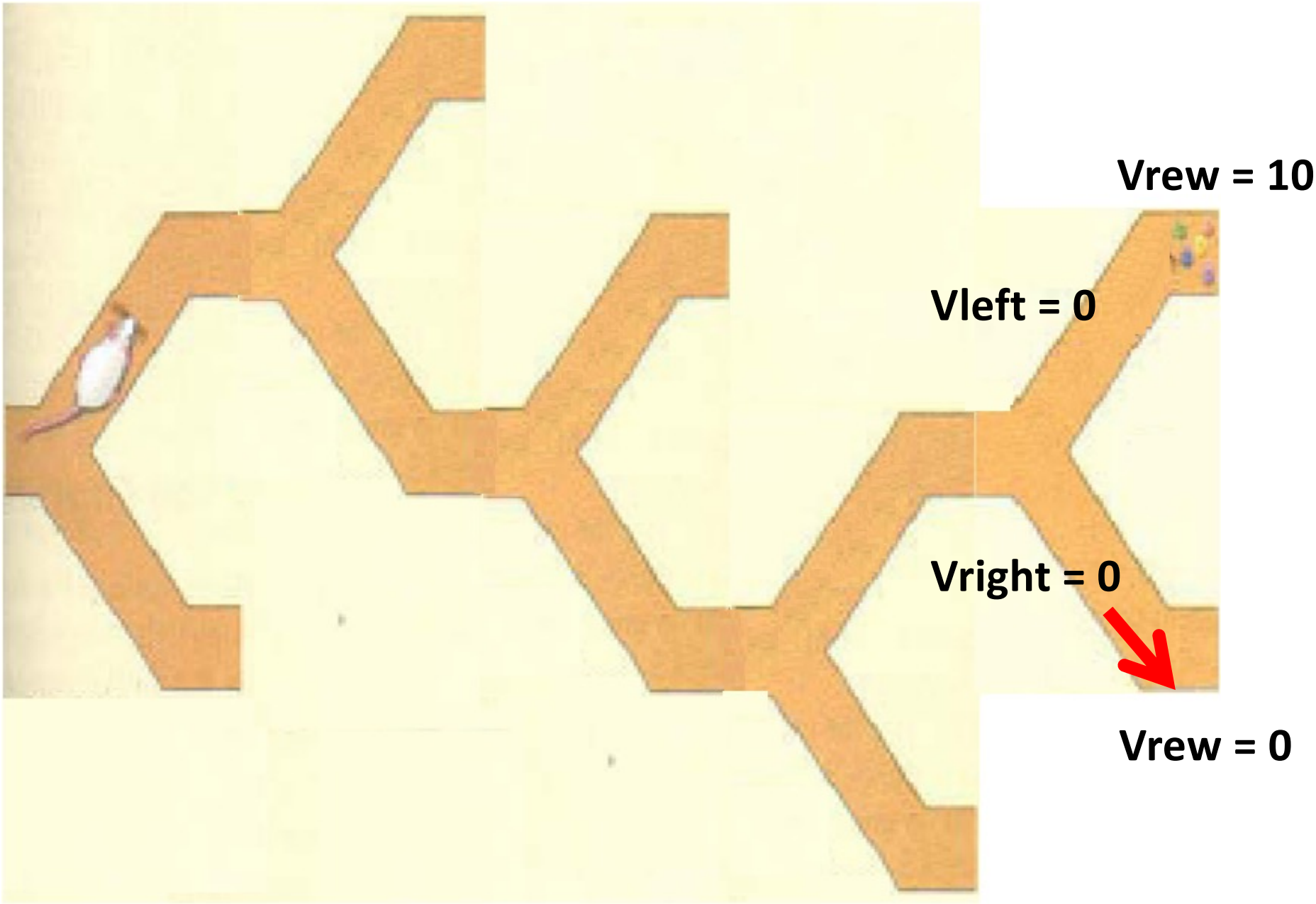
The casino rakes in even more if the stakes are higher:

E(X) = 10(18/38) – 10 (20/38) = -$.53

If the cost is $10 per game, the casino wins an average of 53 cents per game. If 10,000 games are played in a night, that's a cool $5300.


Court or Settle?
100%
75%
30%
$4m
40%
$2m
30%
$1m
25%
$0
100%
$1.2m

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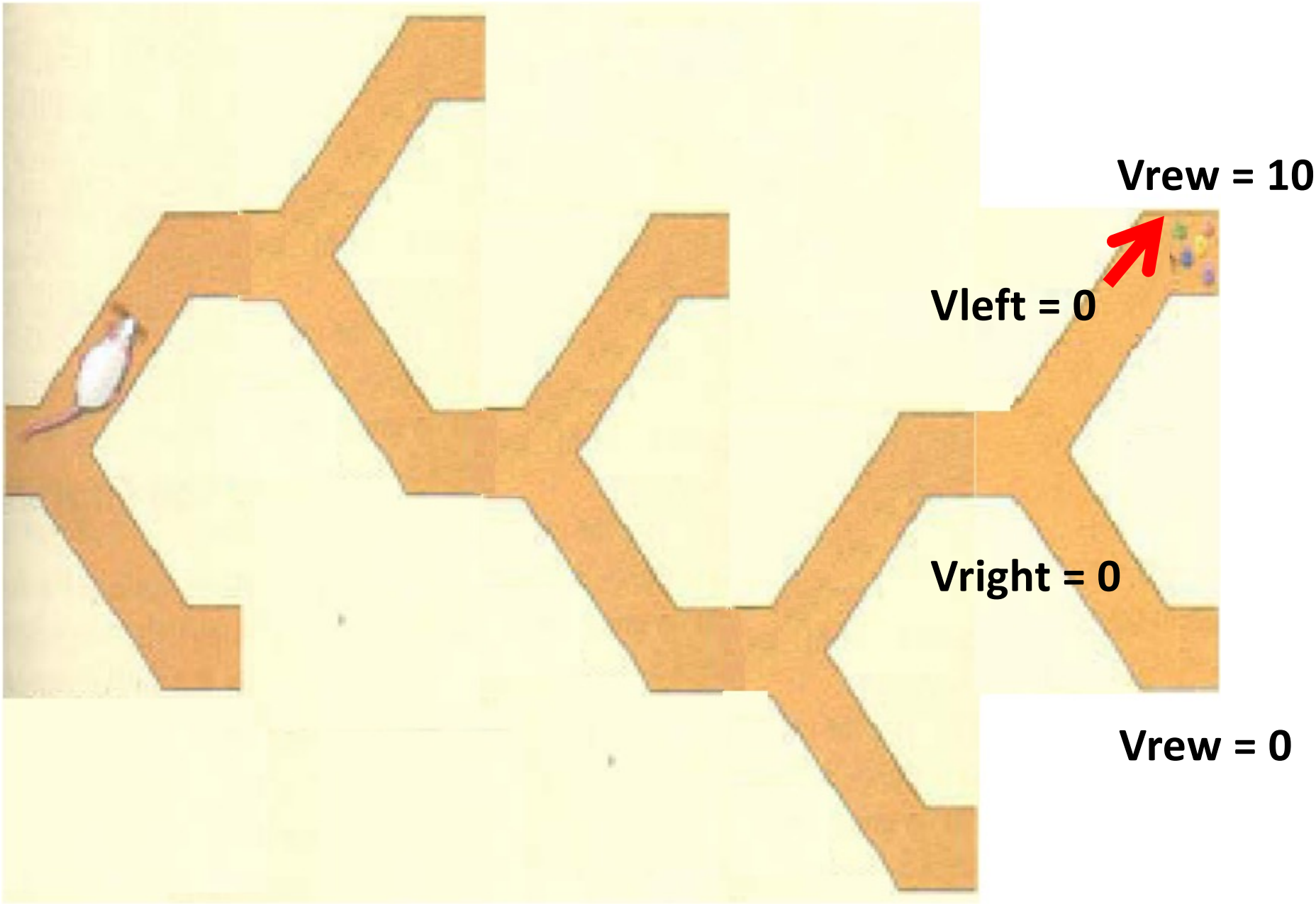
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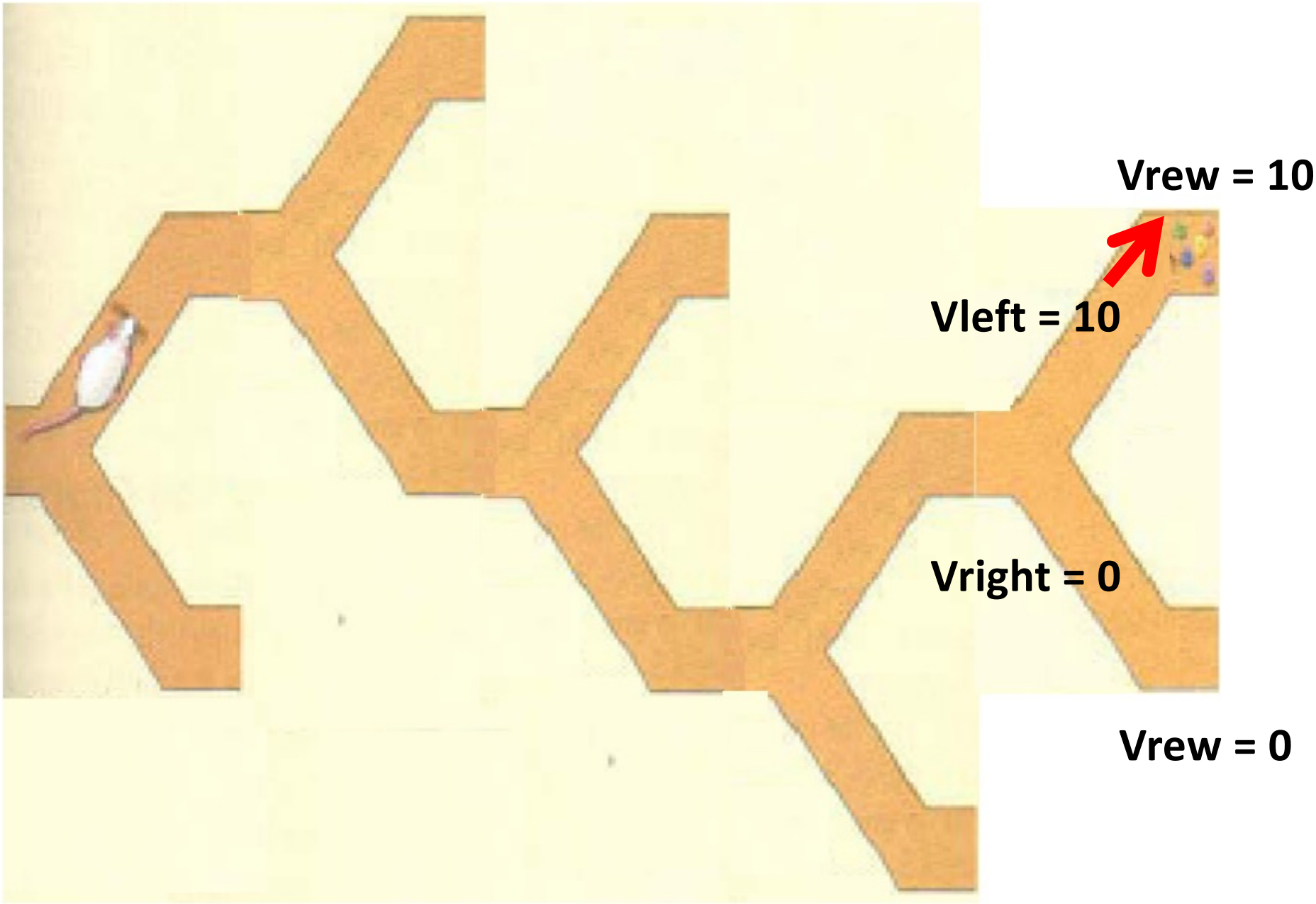
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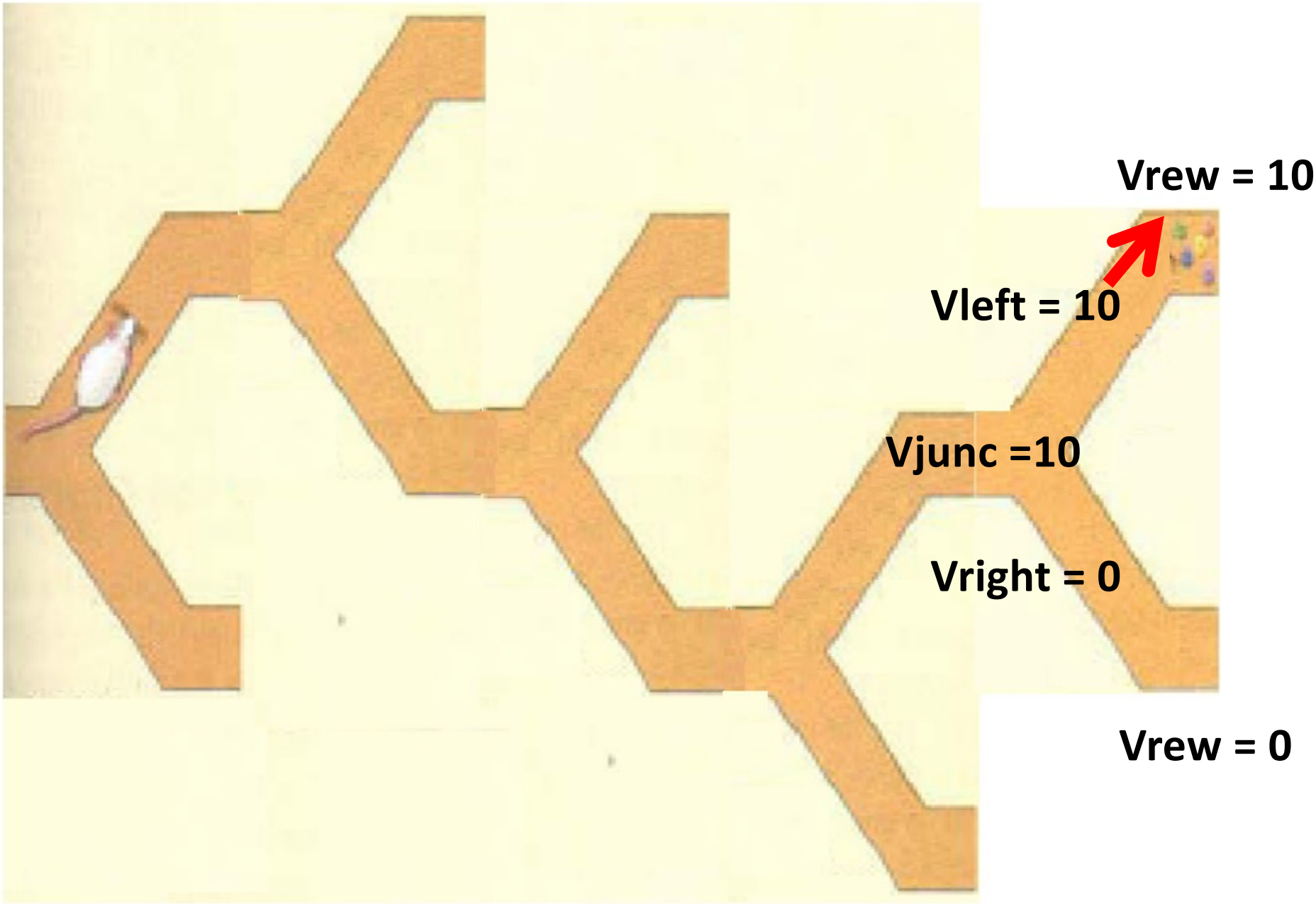
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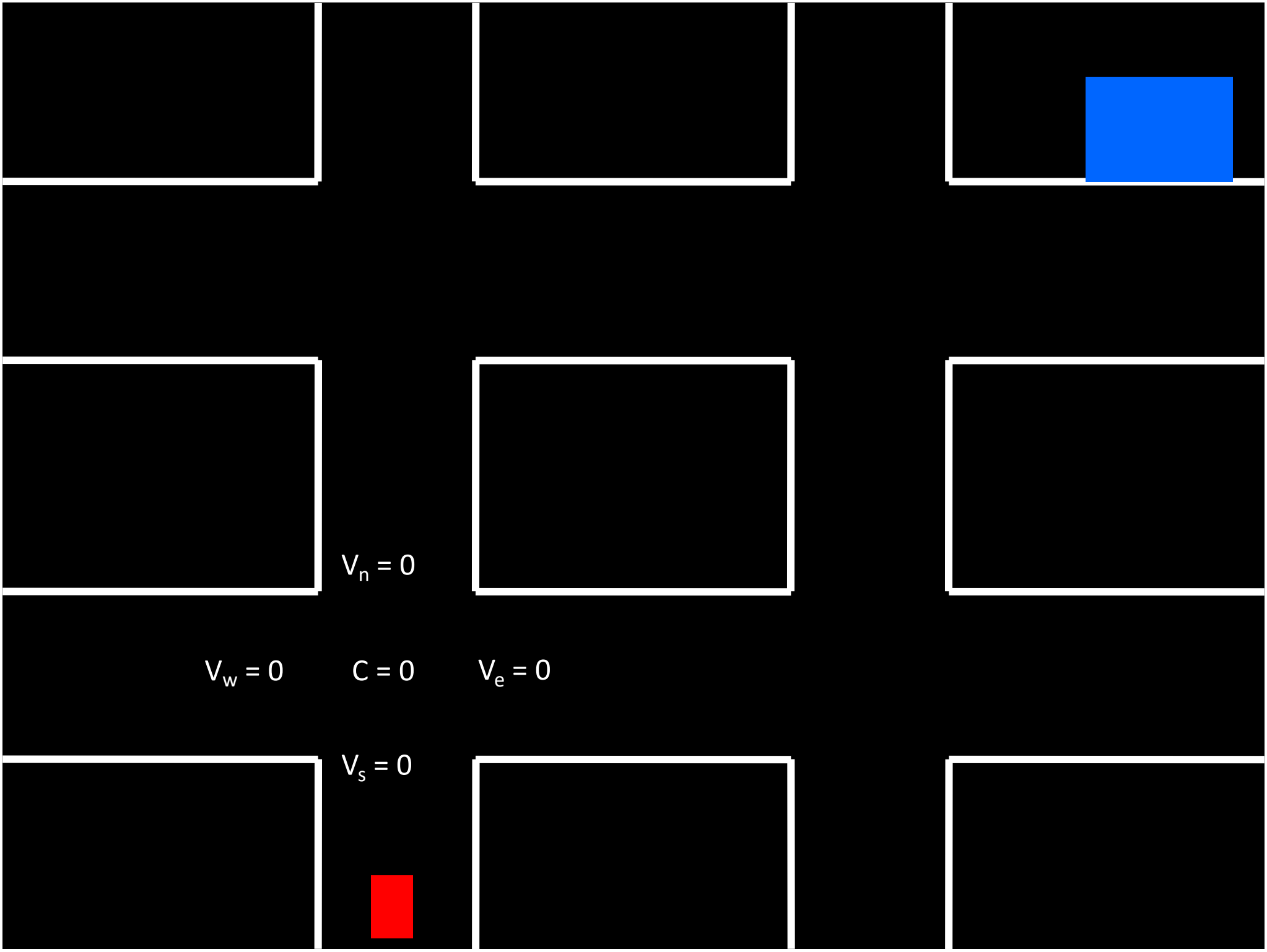
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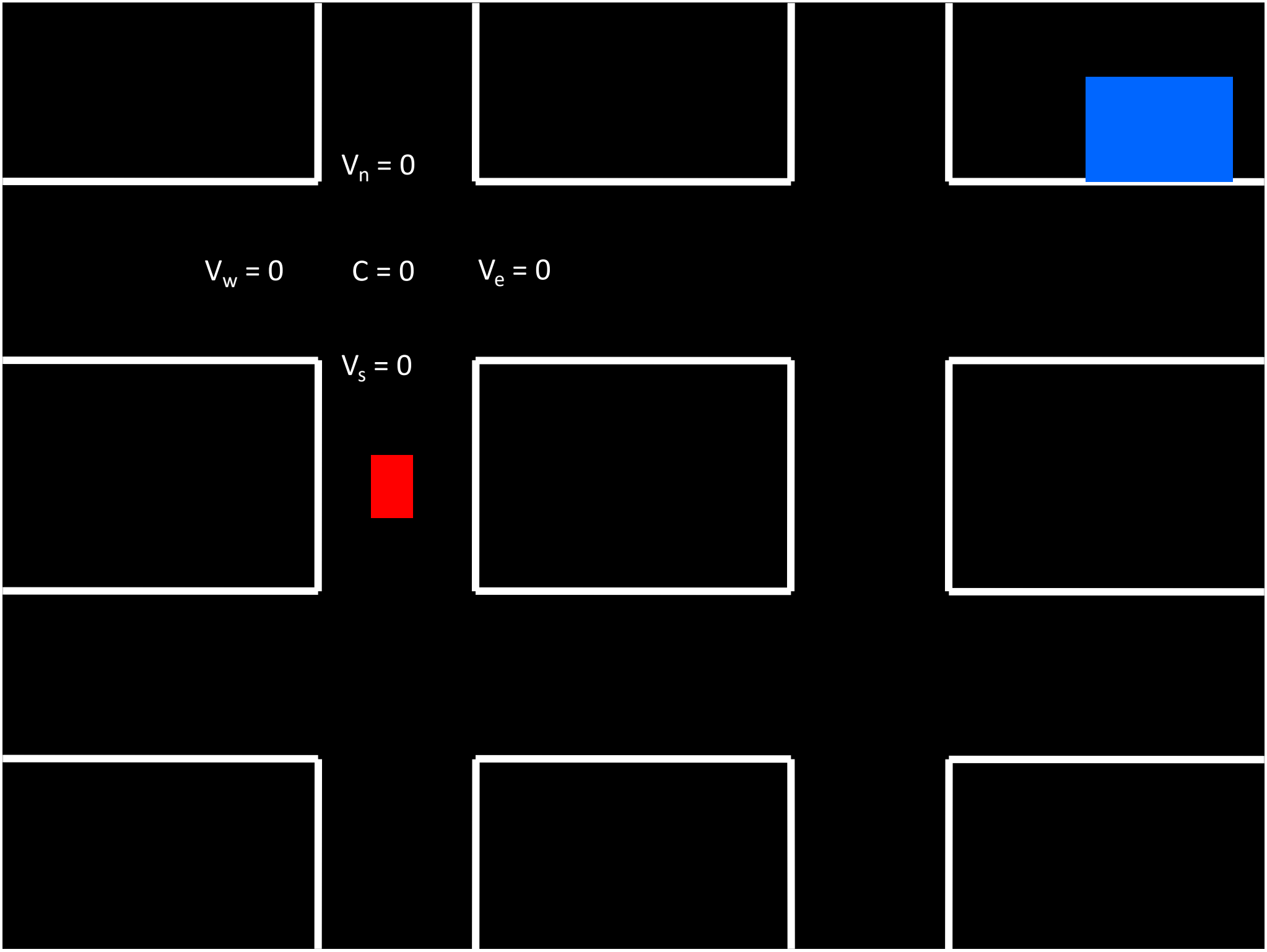
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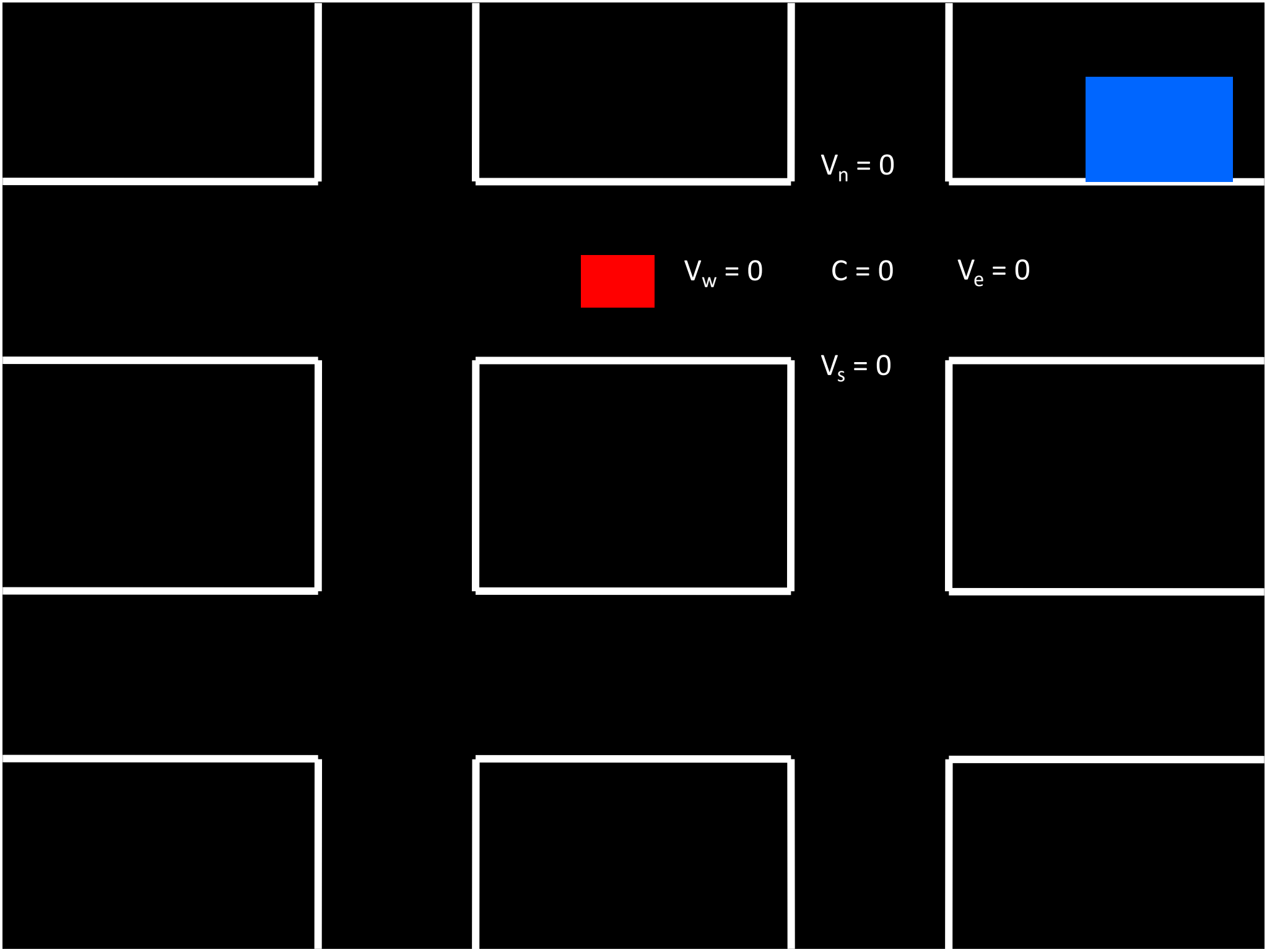
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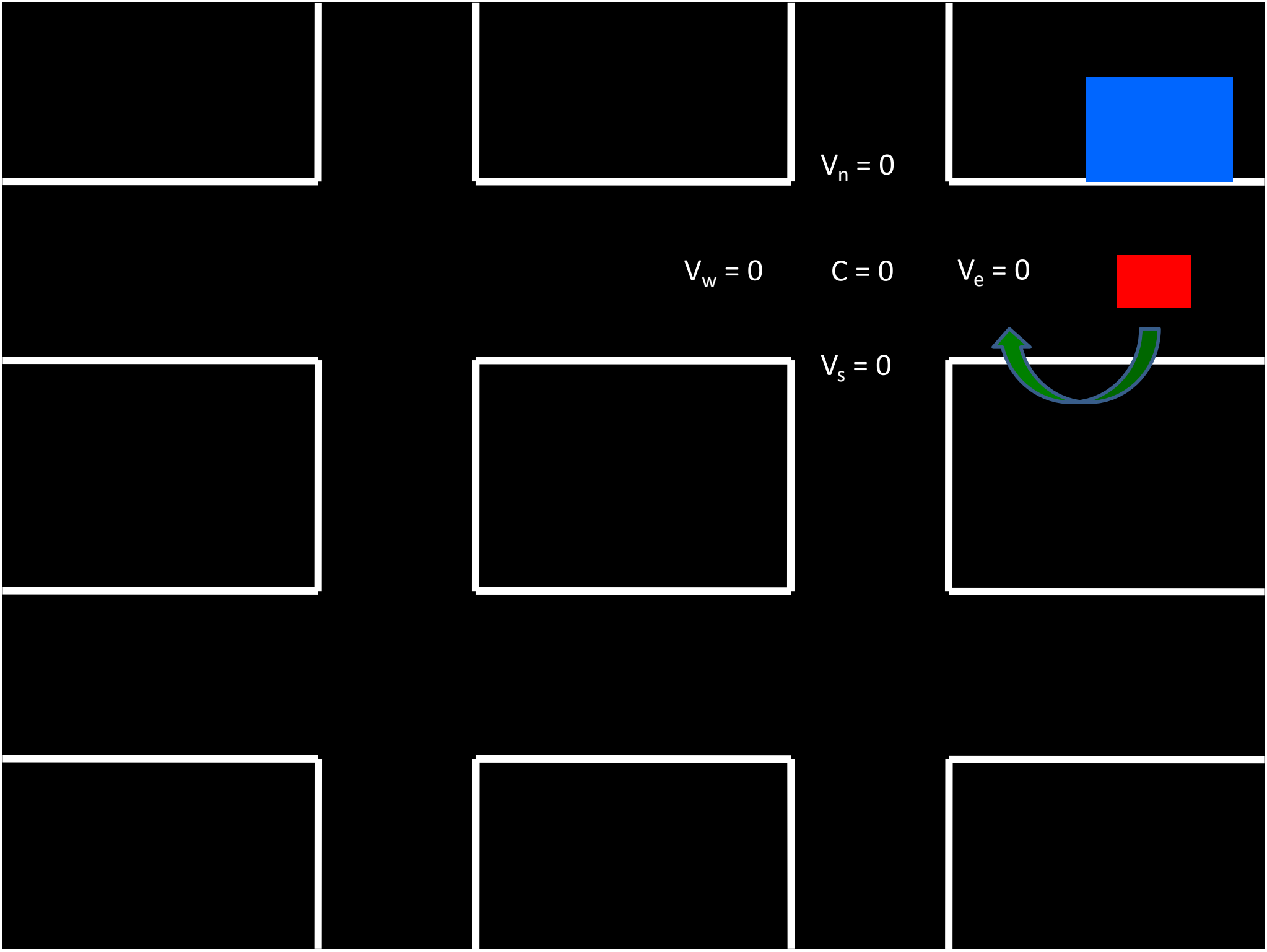
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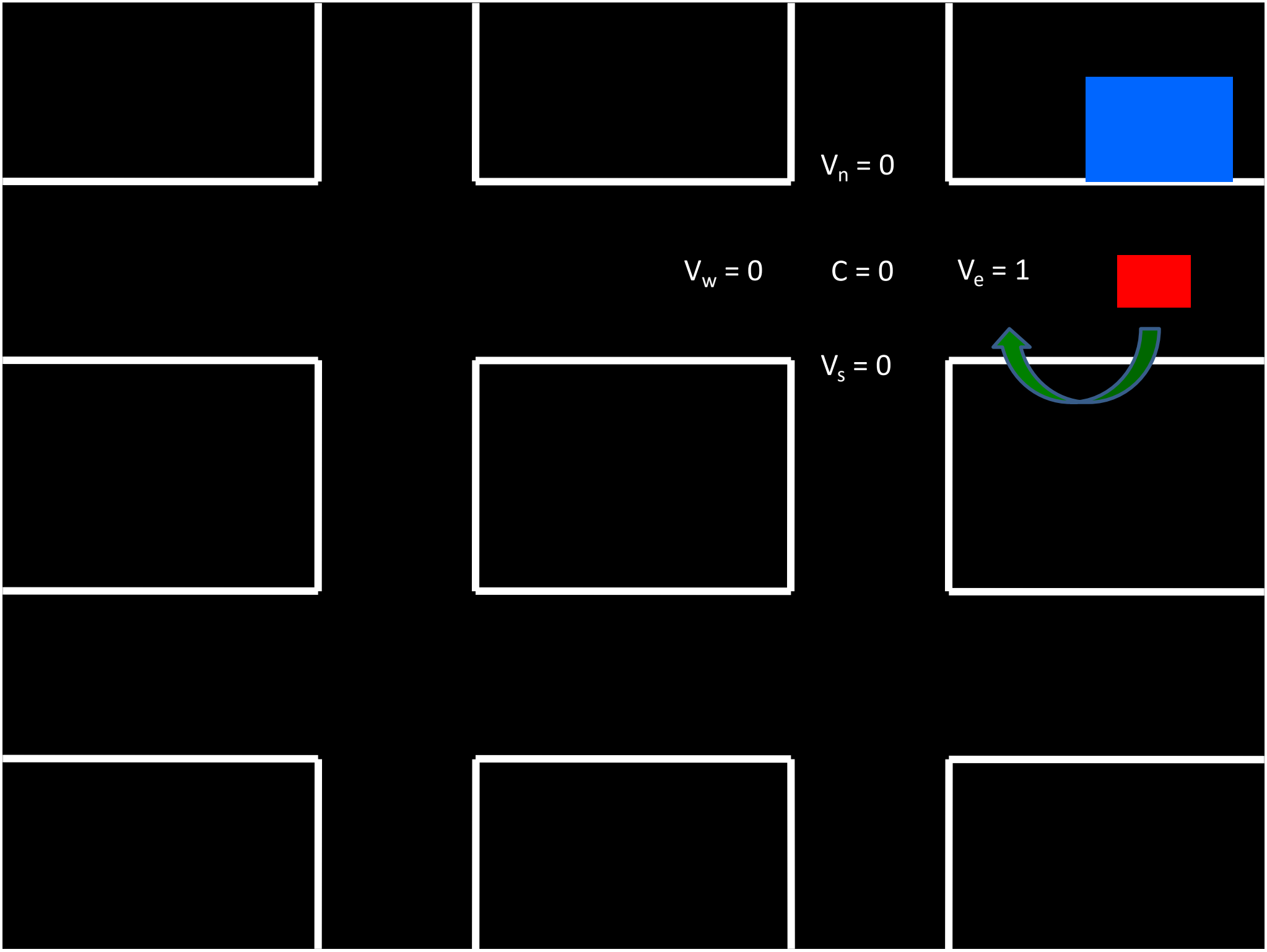
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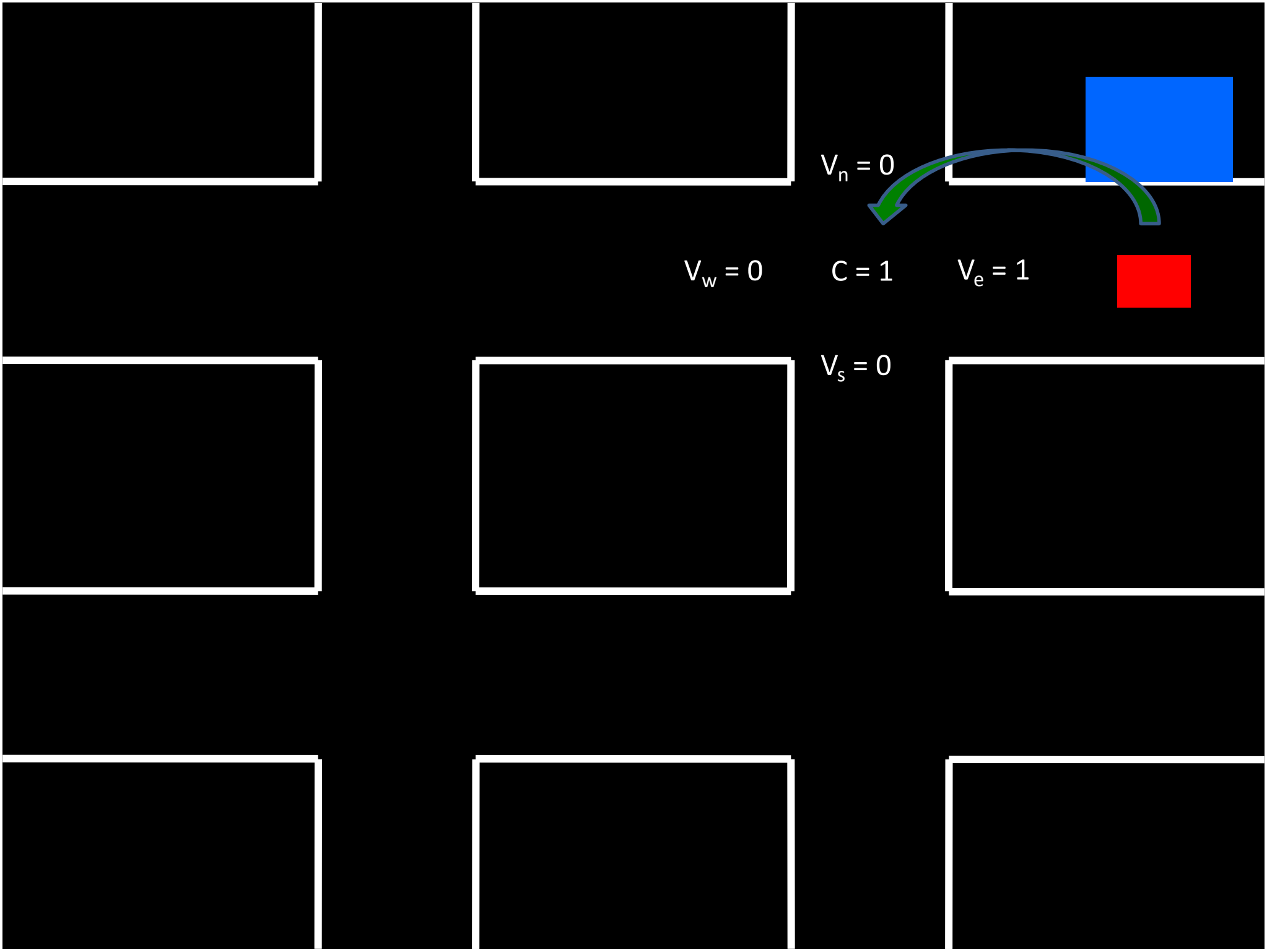
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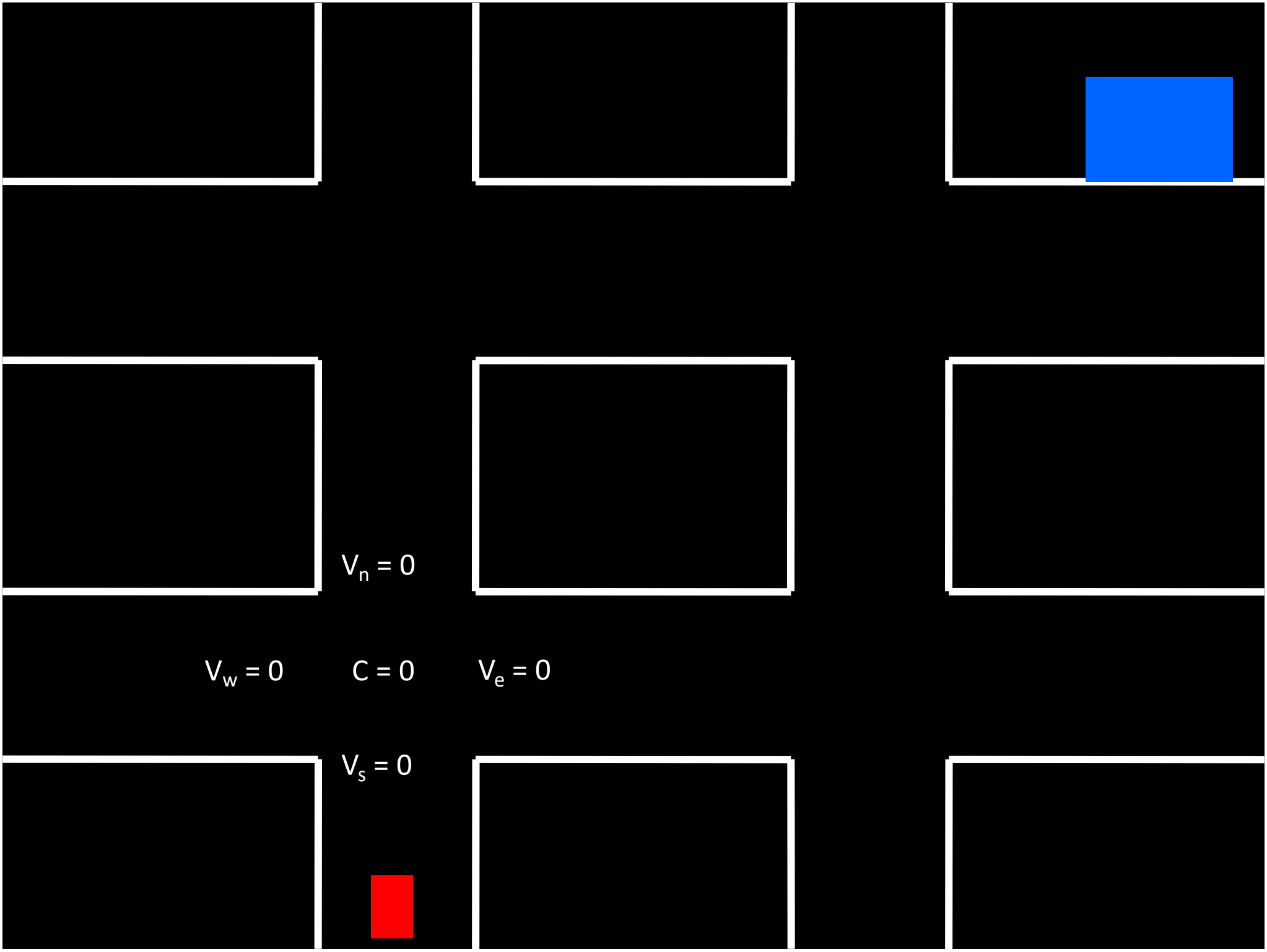
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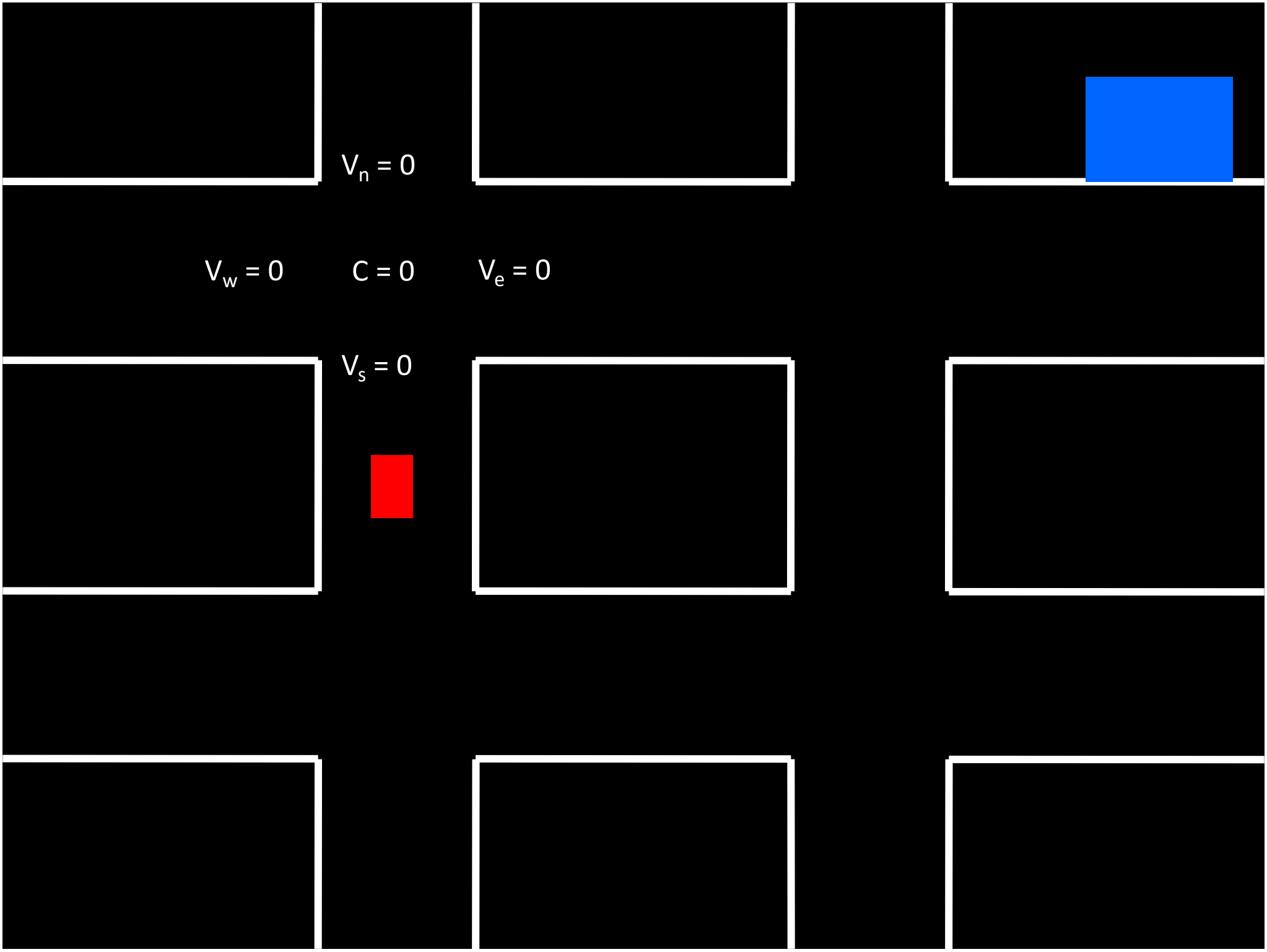
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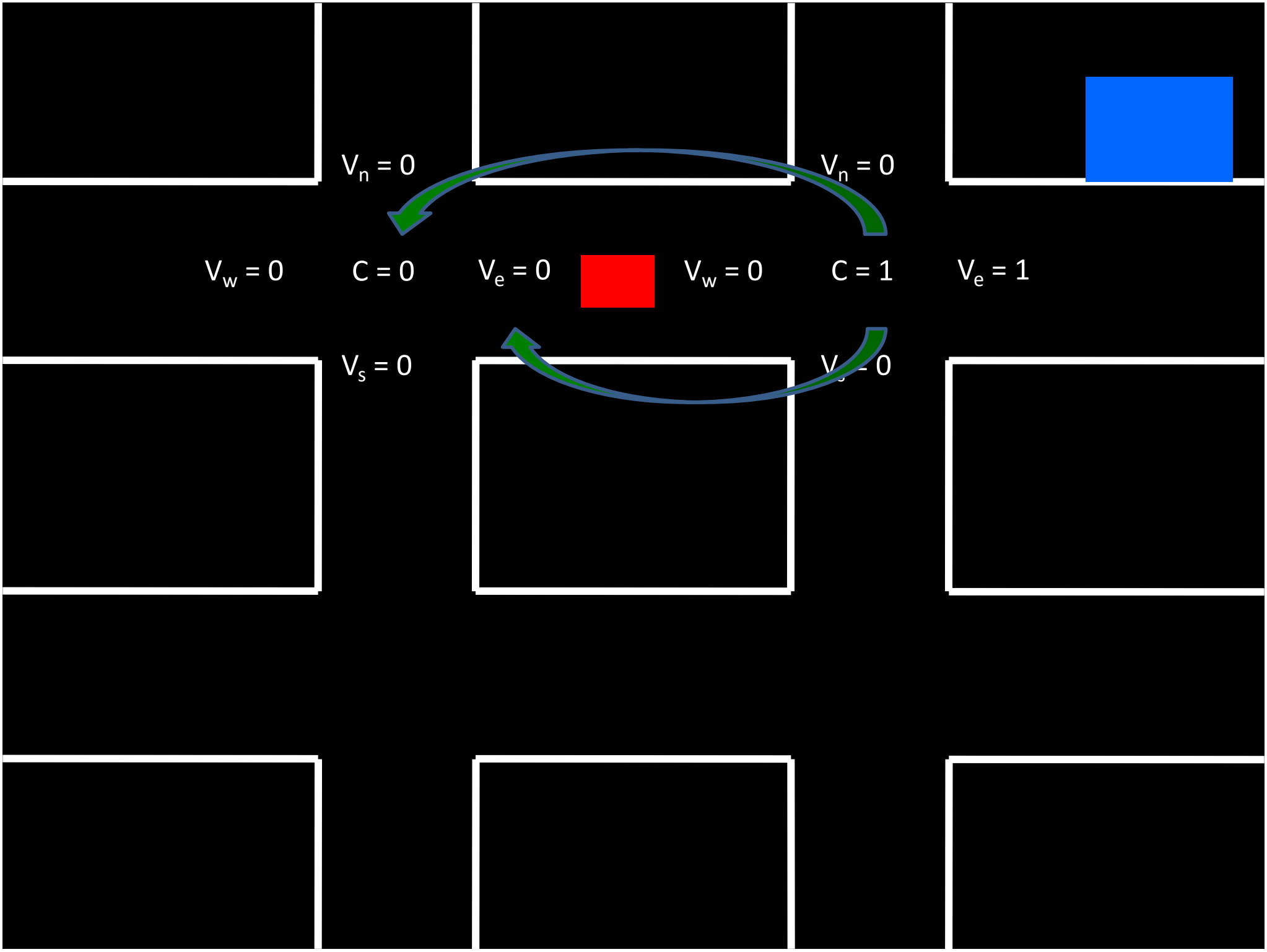
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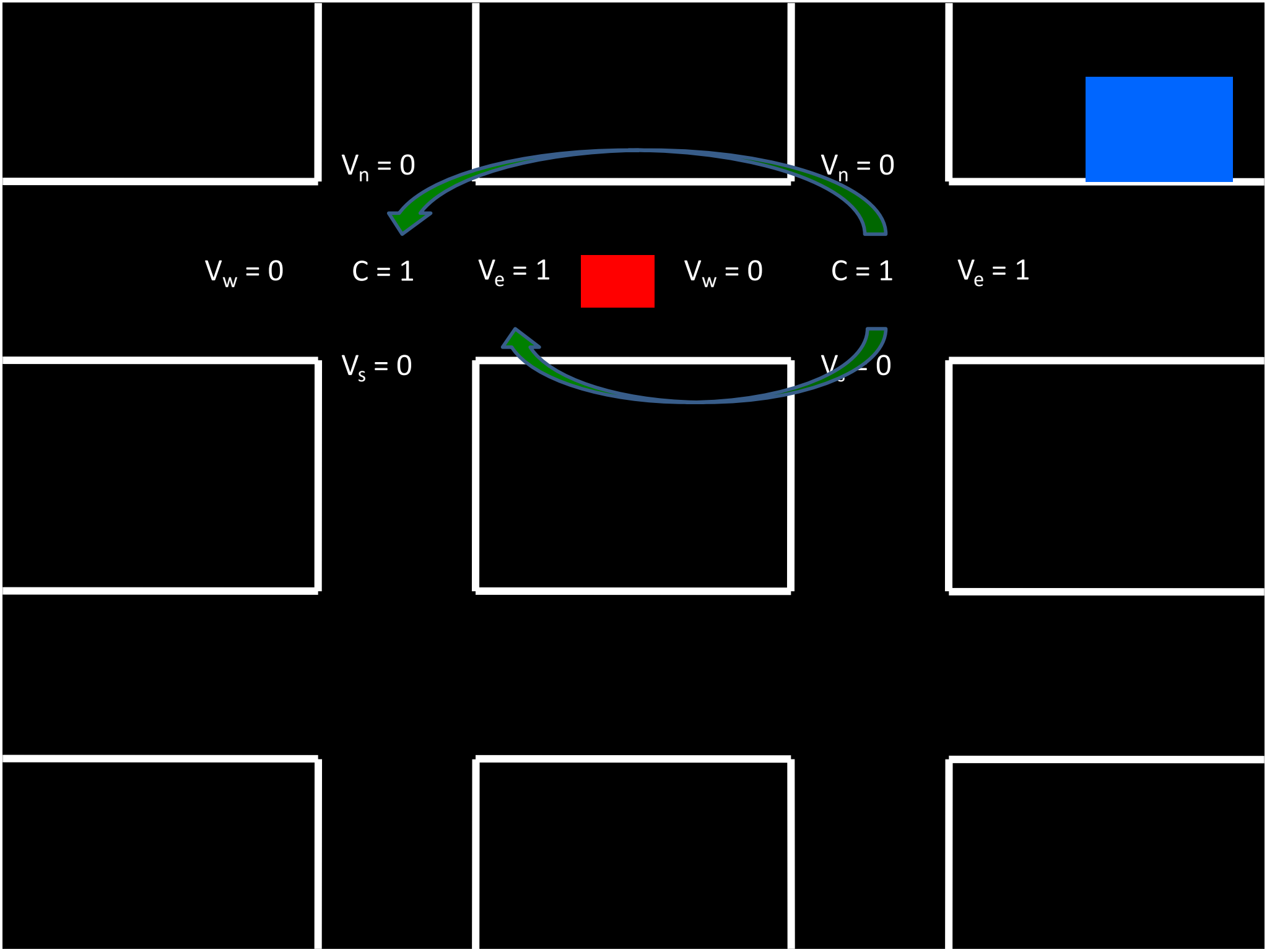
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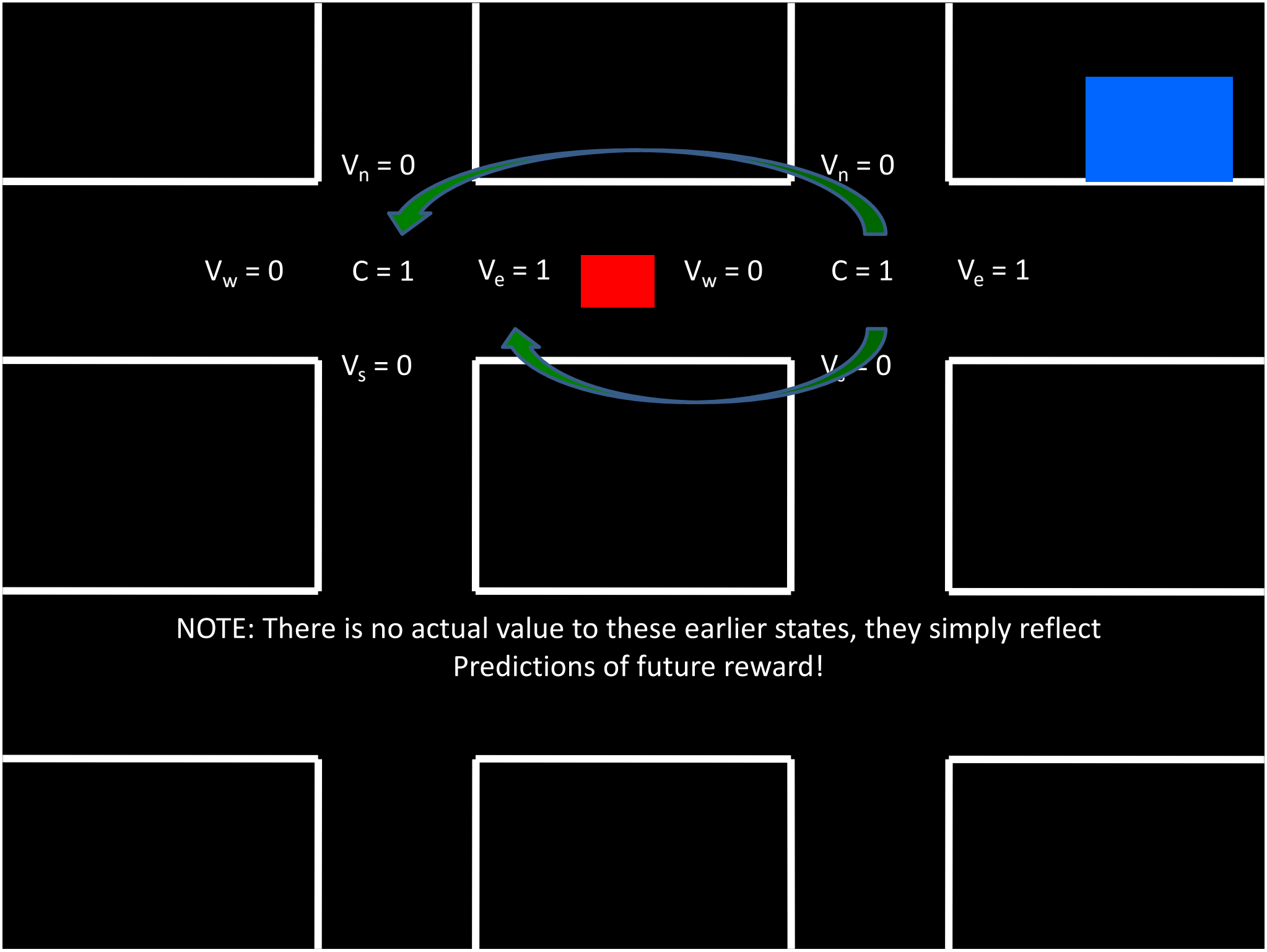


NOTE: There is no actual value to these earlier states, they simply reflect Predictions of future reward!

Tic Tac Toe

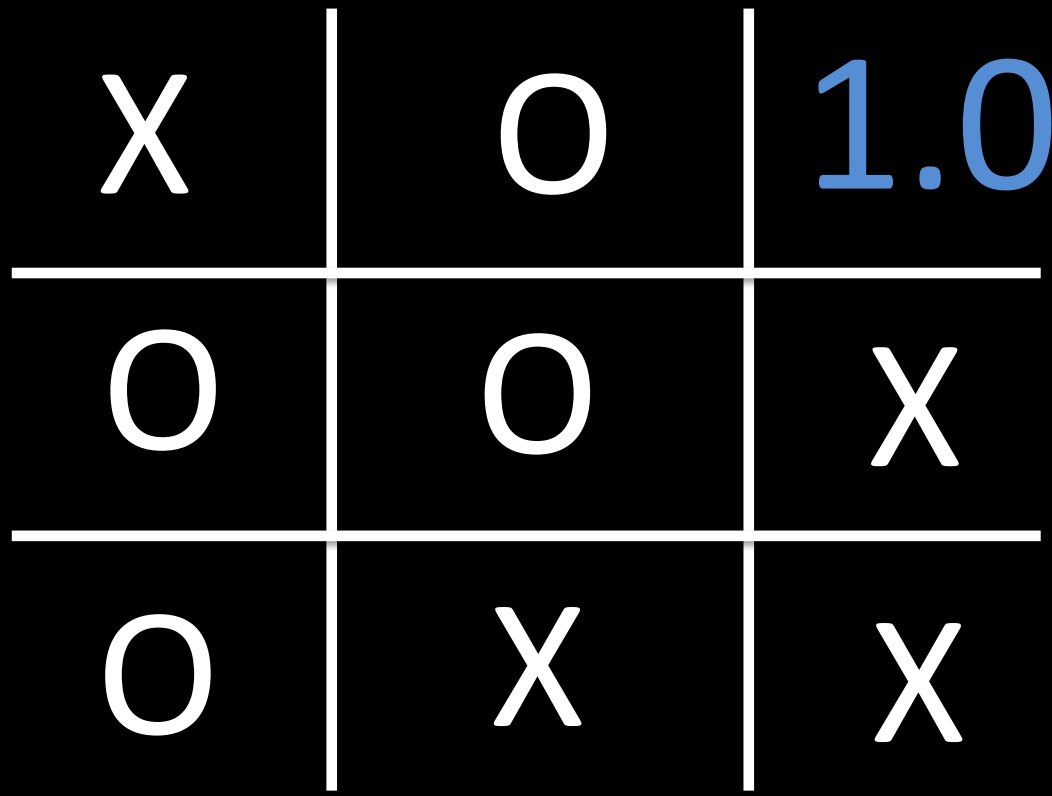
X	0.3	0.5
0.3	O	0.1
0.5	0.1	0.8

What do these values really mean?

Tic Tac Toe

X	O	?
O	O	X
O	X	X

Tic Tac Toe



But is this enough? Eligibility Traces...


STAUNTON

1996 Deep Blue wins a game
1997 Deep Blue wins a match

$p = 0.4345$

$p = 0.4511$

Decision Making

1. Always choose the highest value option

The Problem with Value…


DRIVEGUARD

The Problem with Probability…

What is your chance of winning this Black Jack hand?

Player

Dealer

Rank your chance of dying from…

Car Accident

Bee Sting

Lightning Strike

Plane Crash

Ebola

Terrorist Attack

Shark Attack

1 in 13.3 million chance of **contracting Ebola in America** this year (based on a model of 12 imported cases of Ebola in the course of a year)

1 in 11 million chance of **dying in a plane crash** for an American this year

1 in 9.6 million chance of **dying from a lightning strike** for an American this year

1 in 5.2 million chance of **dying from a bee sting** for an American this year

1 in 3.7 million chance of being **killed by a shark** in your lifetime (worldwide)

1 in 9100 chance of being **killed in a car accident** in America this year

The Problem with Huygens

Prospect Theory

Daniel Kahneman and Amos Tversky

Consider…

Problem 1: In addition to whatever you own, you have been given $1000.
You are now asked to choose of these option
50% chance to win $1000 OR get $500 for sure.

Problem 2: In addition to whatever you own, you have been given $2000.
You are now asked to choose of these option
50% change to lose $1000 OR lose $500 for sure.

Kahneman, 2011

Consider…

Problem 1: In addition to whatever you own, you have been given $1000.

You are now asked to choose of these option

50% change to win $1000 OR get $500 for sure

Problem 2: In addition to whatever you own, you have been given $2000.

You are now asked to choose one of these options:

50% chance to lose $1000 OR lose $500 for sure

Kahneman, 2011

Prospect Theory

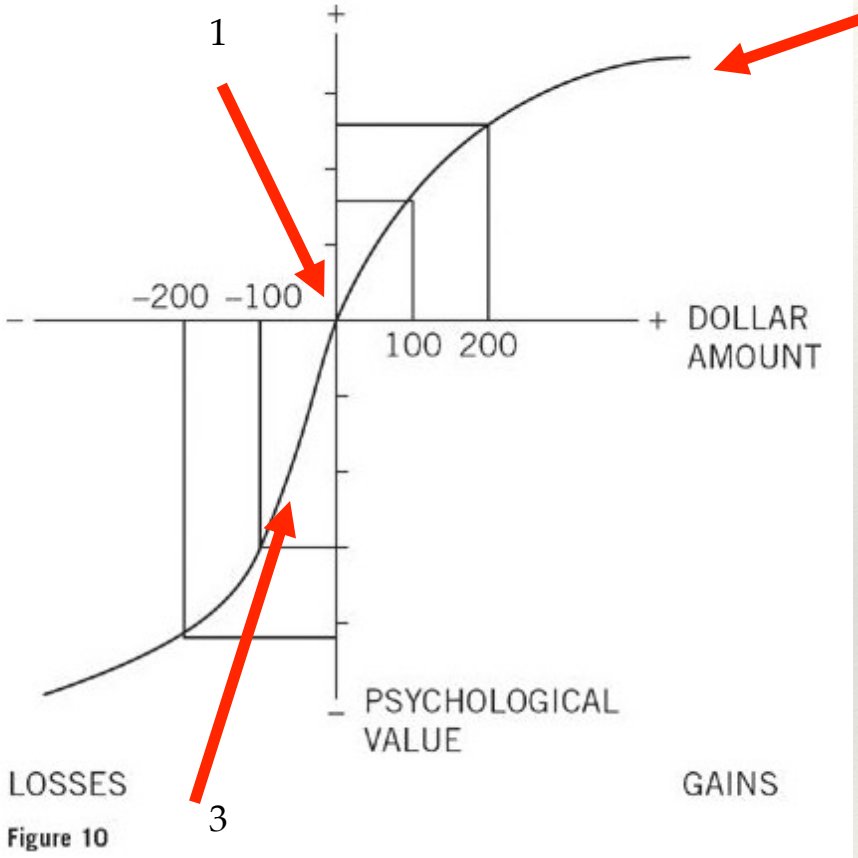


Figure 10

Figure 10

LOSS AVERSION

1. Neutral reference point
2. Diminishing sensitivity to gains and losses
3. *S* is not symmetrical

Explains Loss Aversion

(and other things – Status Quo, Endowment Effect, etc)