EPHE 591: Biomedical Statistics

Bayesian Statistics: Bayes Factors

Breast Cancer Res Treat (2010) 121:671–678
DOI 10.1007/s10549-009-0604-x

EPIDEMIOLOGY

Evaluation of breast cancer service screening programme with a Bayesian approach: mortality analysis in a Finnish region

Browse Journals & Magazines > Biomedical Engineering, IEEE ...> Volume:59 Issue:5

A Boosted Bayesian Multiresolution Classifier for Prostate Cancer Detection From Digitized Needle Biopsies

Advances in Artificial Intelligence - IBERAMIA-SBIA 2006
Lecture Notes in Computer Science Volume 4140, 2006, pp 622-631

Bayesian Model Combination and Its Application to Cervical Cancer Detection

Miriam Martínez, Luis Enrique Sucar, Hector Gabriel Acosta, Nicandro Cruz

More Problems with P

Frequentist Approach

p(Data/Ho)

”How likely are the data if we assume the null hypothesis is true”

Bayesian Approach

p(Ho/Data)

“How likely is the null hypothesis given the data”

The p - value does not answer p(Ho/Data), even though a lot of people think it does.

p(pregnant/female) versus p(female/pregnant)

How viable is our hypothesis given our data?


LIBERTY
IN GOD WE TRUST
1932

10


UNITED STATES OF AMERICA
E PLURIBUS UNUM
QUARTER DOLLAR

4

Frequentist Approach

10/14

Probability of 1 head next:

= 0.714

0.714 X 0.714

Probability of 2 heads next:

= 0.51

THIS IS CLEARLY WRONG

Bayesian Approach

Rev. T. Bayes (1707 - 1761)

Posterior

Prior

Likelihood

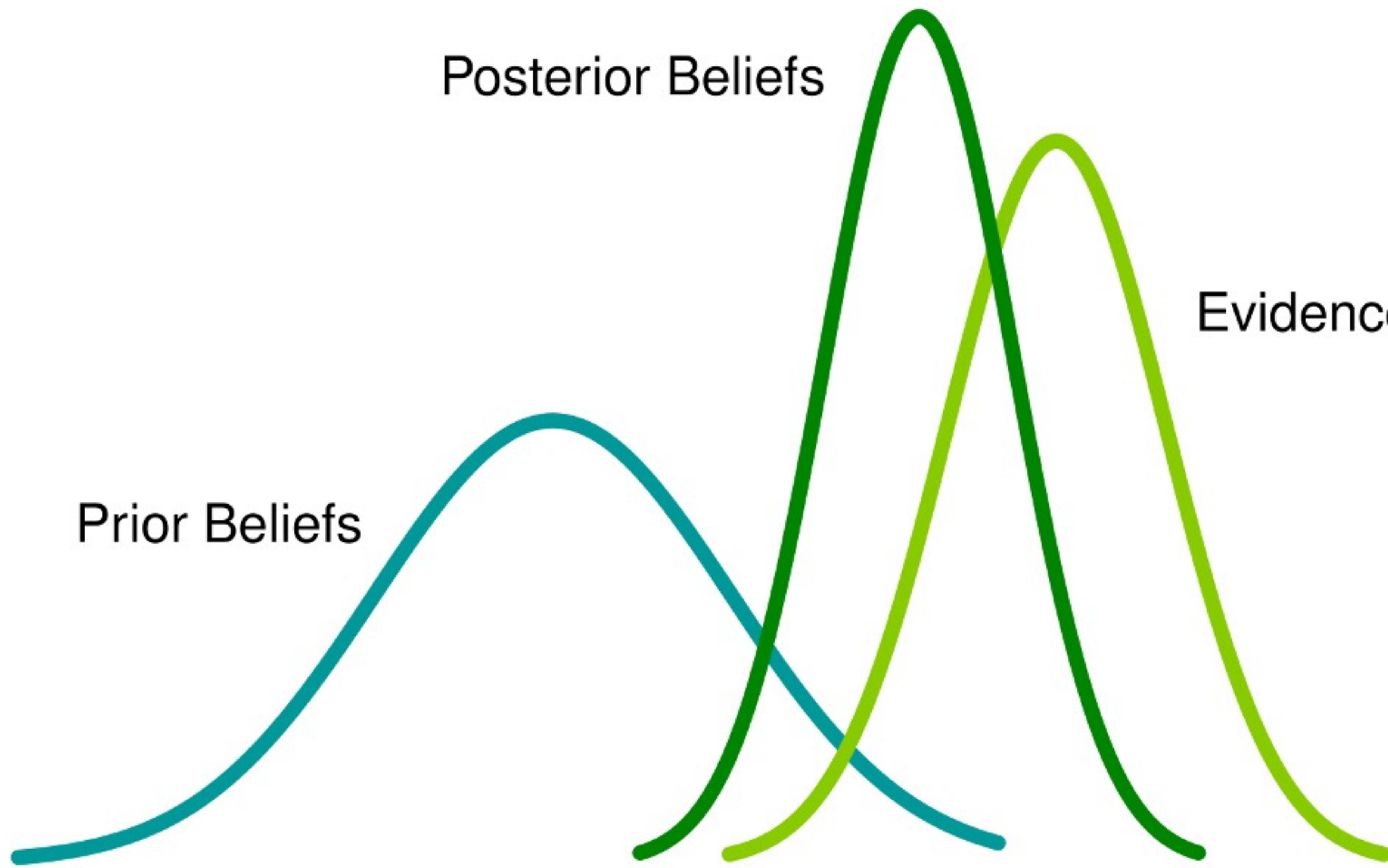
$$P(H|E) = \frac{P(H)P(E|H)}{P(E)}$$

Normalising Constant

10 **4**

Bayesian Logic would compute the odds of the next flip being heads as 50%

Bayesian methods generate conclusions using statistics generated from the data.


Posterior Beliefs
Evidence
Prior Beliefs

USING BAYESIAN STATISTICS: THE BAYES FACTOR

The Bayes Factor

$$\frac{p(Ho/Data)}{p(H1/Data)} = \frac{p(Data/Ho)}{p(Data/H1)} \times \frac{p(Ho)}{p(H1)}$$

posterior odds   Bayes Factor   prior odds

Typically, we assume the prior odds the same – both hypotheses are equally likely so that terms goes to 1.

The Bayes Factor

The Bayes Factor reflects a change in the prior odds based on the data.

Essentially, it will tell you the relative likelihood of Ho relative to H1.

Because the prior odds are 1, the Bayes Factor becomes the posterior odds.

Bayes Factor Logic

Prior

Ho = H1 Both hypotheses are equally likely

Bayes Factor

Collect some data, compute Bayes Factor

Posterior

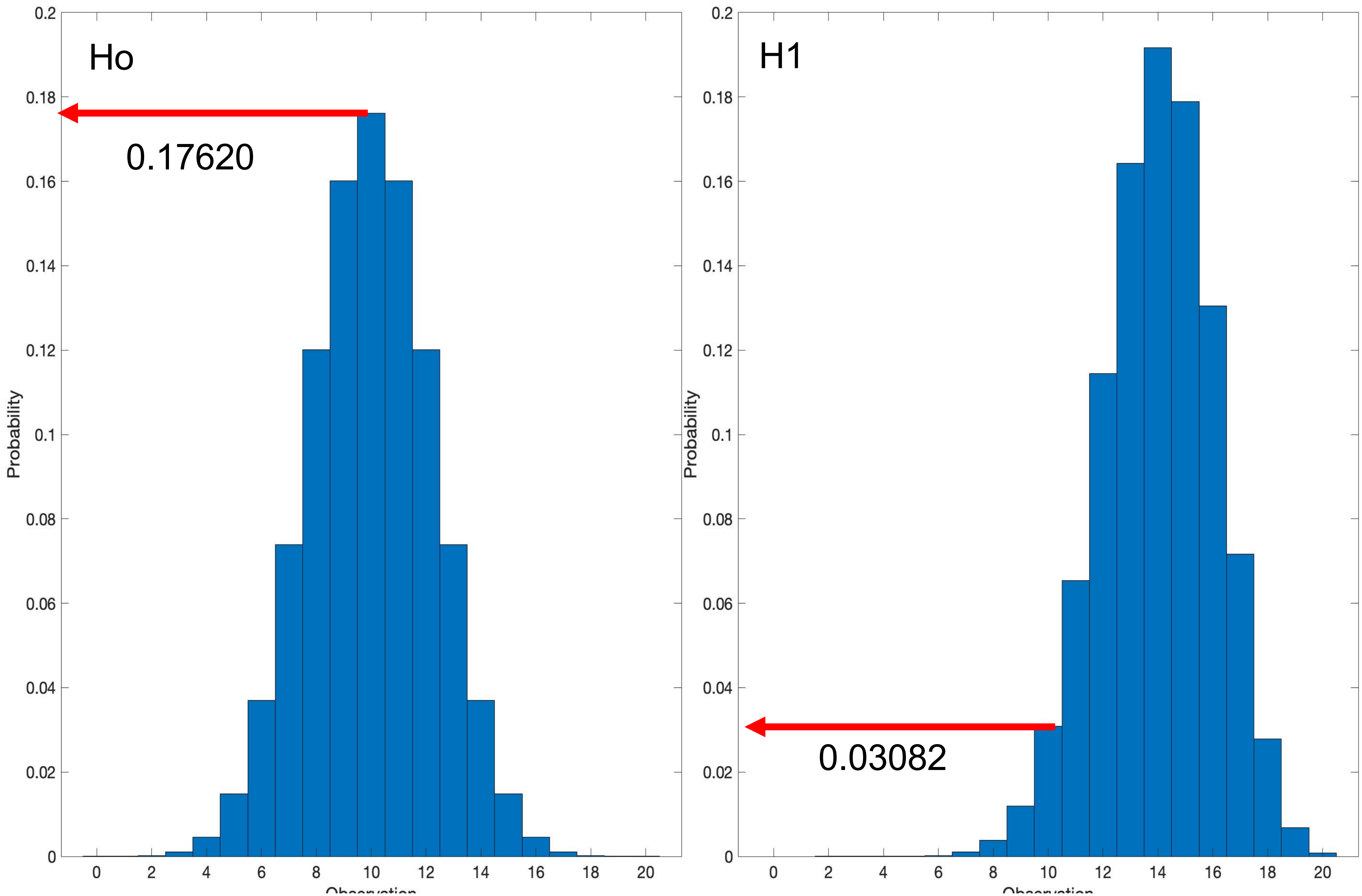
Determine likelihood of Ho relative to H1

Example

Let's say you are going to flip a coin 20 times. You have begun to think you are really good at calling heads, so you think you can do better than chance.

Ho	pr = 0.5	NOTE! This is our PRIOR. Or, our PRIOR ODDS.
H1	pr = 0.7	

So, you flip the coin 20 times and get 10 heads.



Ho
0.17620
Probability
Observation
H1
0.03082
Probability
Observation

Computing the Bayes Factor

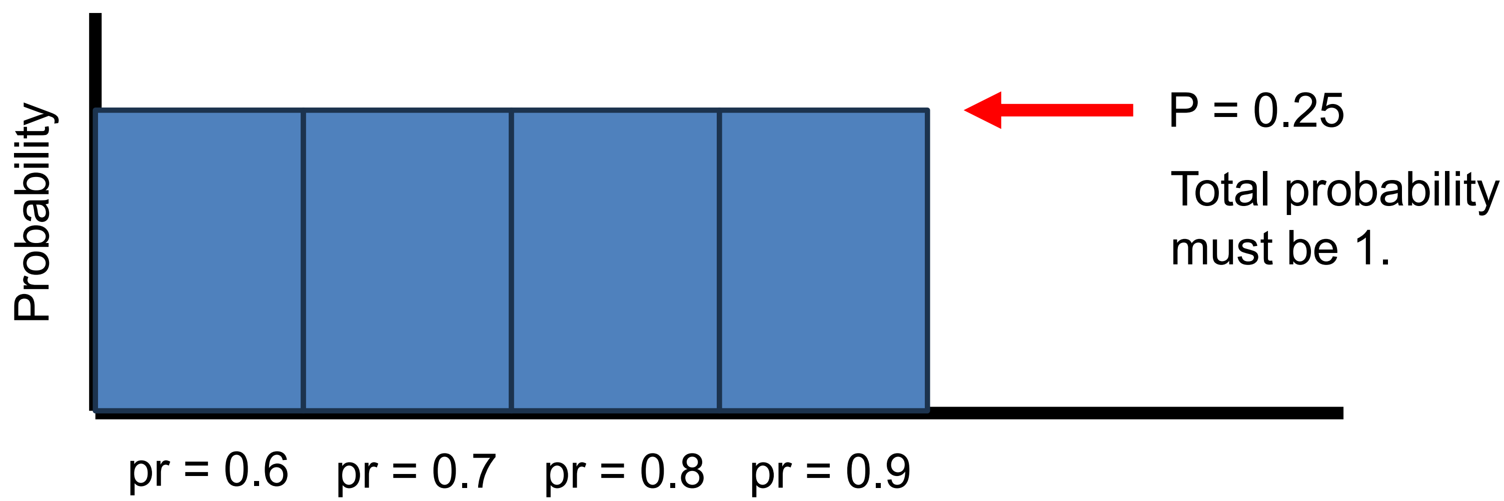
$$BF_{01} = \frac{p(Data/Ho)}{p(Data/H1)} = 0.17620 / 0.03082 = 5.717$$

How do we interpret this?

Ho is 5.717 times more likely than H1

But what if our prior is not a constant value?

Let's take the same example, but assume that H1 just means better than chance (pr = 0.6, 0.7, 0.8, 0.9). So, in the first example the prior was pr = 0.6 but what do we do if it is a range of numbers? We use a DISTRIBUTION.

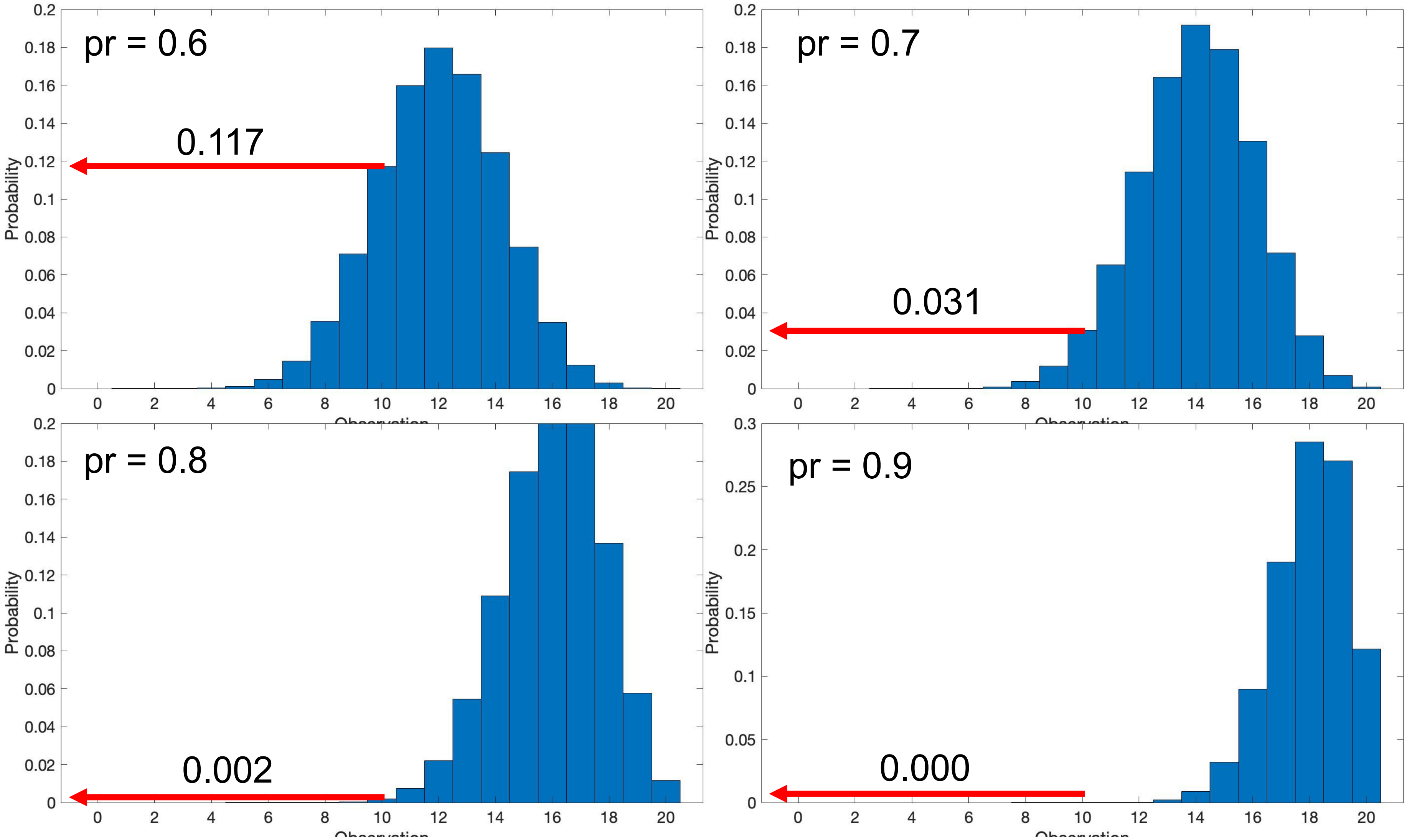


But what if our prior is not a constant value?

Computing the Bayes Factor is more tricky.

You need to know p(Data/H1) for each value of pr.

Recall, the DATA – the data was 10 heads out of 20 flips



Computing the Bayes Factor

P(Data/H1) = 0.117 * 0.25 + 0.031 * 0.25 + 0.002 * 0.25 + 0.000 * 0.25

P(Data/H1) = 0.0375

P(Data/Ho) = 0.1762 (as before)

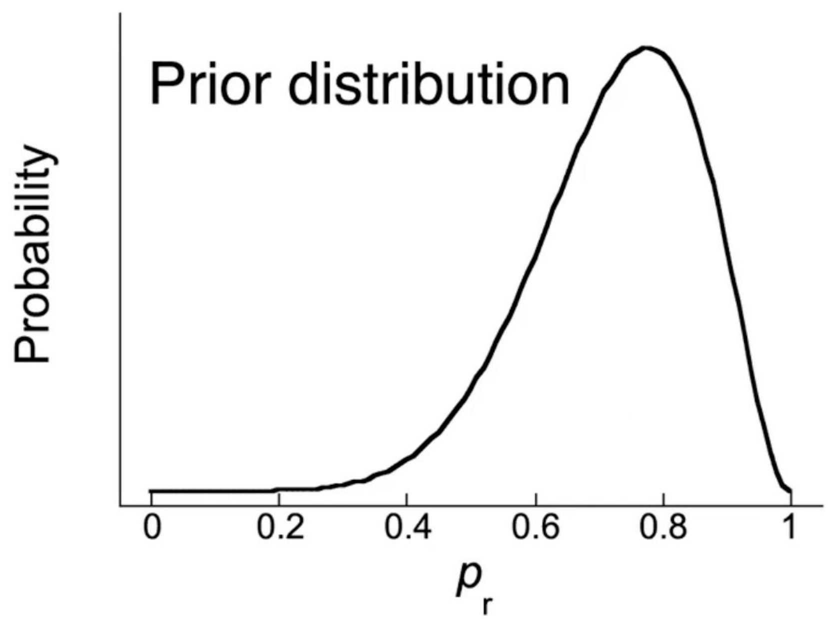
Bayes Factor = 0.1762 / 0.0375 = 4.699

Ho is 4.699 times more likely than H1 given the data.

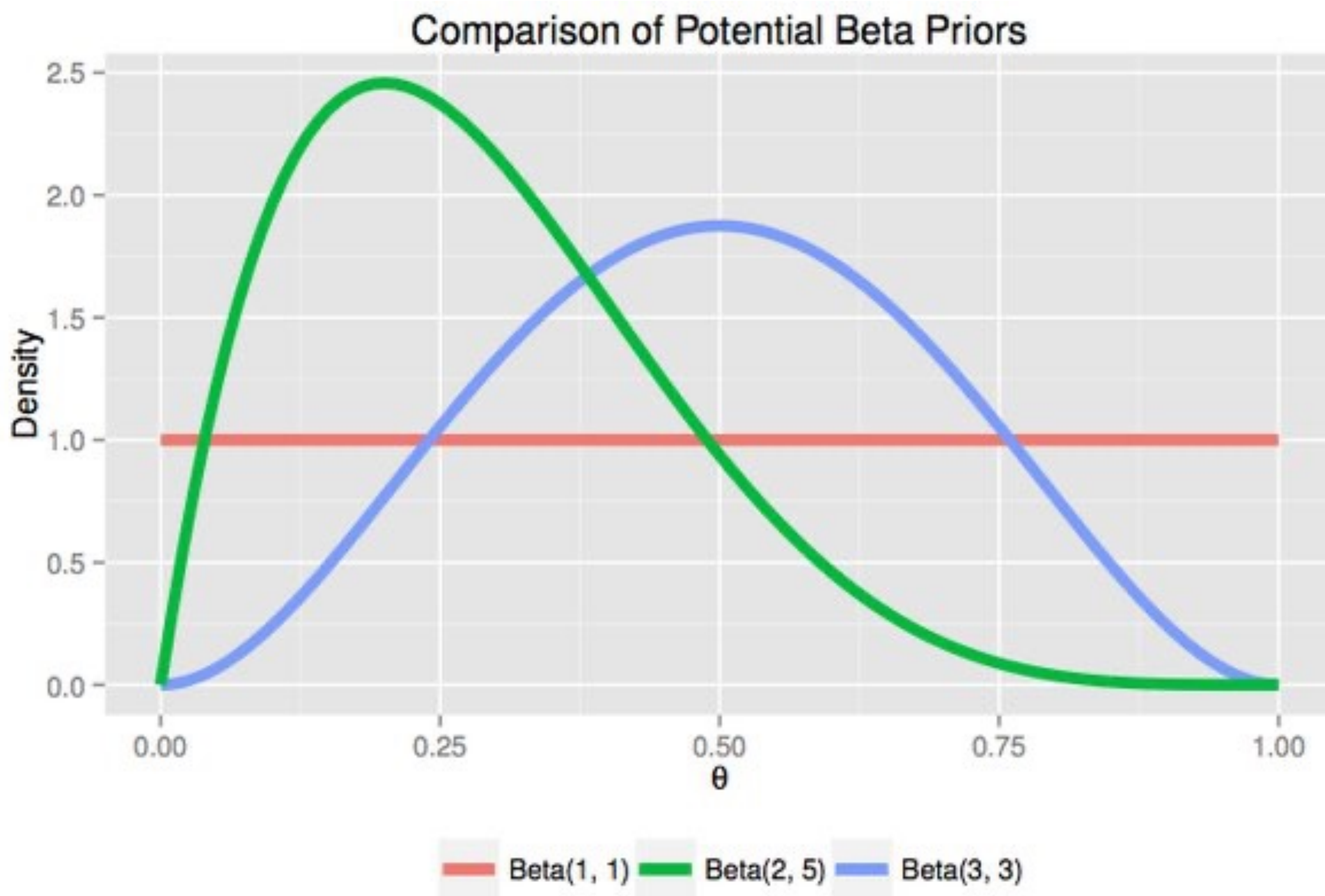
But…

What about pr = 0.67, pr = 0.73, etc…

We use distributions as priors as opposed to numbers. For example:



Prior Distributions



Interpreting the Bayes Factor

Statistic		Support for H_1	
Bayes Factor	**Inverse of Bayes Factor**	**Raftery**	**Jeffreys**
1–.33	1–3	Weak	Anecdotal
.33–.10	3–10	Positive	Substantial
.10–.05	10–20	Positive	Strong
.05–.03	20–30	Strong	Strong
.03–.01	30–100	Strong	Very Strong
.01–.0067	100–150	Strong	Decisive
<.0067	>150	Very Strong	Decisive

Other Thoughts

Your choice of prior matters (sometimes)

JASP will give you BF_{10} not BF_{01} (the inverse)

Some stats programs give you the Ln(BF)

Credit

Credit to Mike Masson for all of this material and information!